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Some Aspects of Symmetry Breaking in Unified Weak-Electromagnetic Gauge Theories

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SOME ASPECTS OF SYMMETRY BREAKING IN UNIFIED WEAK-ELECTROMAGNETIC GAUGE THEORIES

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PREFACE

It is a pleasure to thank those who have helped me along the road from student to card-carrying physicist. In particular I would like to thank my advisor, Professor Pais, who shared with me his enthusiasm and deep understanding of the beauty and subtlety of the subject. His encouragement and helpful suggestions were indispensable for the completion of this work. I would also like to thank the other members of the physics group, in particular Professors Bég, Khuri and Pagels, who imparted to me in the course of many discussions the flavor of their taste for physics.

ABSTRACT

In this thesis, we focus on some problems of symmetry breaking in unified weak-electromagnetic gauge theories. In Chapter 1 we set the scene with a brief history of weak interaction theory up until the impasse which led to the development of the unified weak-electromagnetic gauge theory strategy. In Chapter 2 we describe the basic ideas underlying the new gauge strategy, illustrate how these ideas can be concretized in a specific model and discuss some of the prospects and problems which remain to be solved.

In Chapters 3 and 4 we make a small contribution towards some of the problems which arise in applying the gauge strategy. We focus in particular on the role of the Higgs scalars in the spontaneous breakdown of the theory. In Chapter 3, we consider the following question: how can we break the gauge symmetry in such a way that all of the weak vector mesons acquire mass but the photon remains massless? In Chapter 4, in the context of a specific model, we study the effects on calculable quantities, such as the proton-neutron mass difference, of varying the Higgs content and investigate the appearance of pions as part of the Higgs system.

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1. INTRODUCTION

I. A Brief History of Weak Interaction Theory

The story of weak interaction theory begins in 1934 with Fermi's attempt to describe the β decay of nuclei.[1] Although nuclear β decay had been discovered around the turn of the century by Becquerel, the necessary ingredients for a theoretical explanation were not available until Chadwick's discovery of the neutron in 1932[2] and Pauli's hypothesis of the neutrino in 1930.[3] Pauli's hypothesis of a neutral unobserved particle of zero or nearly zero mass was necessary to maintain the principle of energy conservation in β decay since the electrons were emitted with a spread of energies. Thus nuclear β decay was interpreted as the transformation of a neutron in the nucleus into a proton with the simultaneous emission of an electron and "neutrino". (By current convention the neutrino is actually an antineutrino): $n \rightarrow p + e^- + \bar{\nu}_e$.

Fermi hypothesized that the interaction responsible for β decay is a simple four point direct coupling without derivatives of the form

$$H_\beta = G_\beta \bar{\psi}_p(x)\, O \psi_n(x) \bar{\psi}_e(x)\, O \psi_{\nu e}(x)$$

Relativistic invariance allows a certain amount of freedom in the choice of the operator O; it may be scalar ($O_s=1$), pseudoscalar ($O_p=\gamma_5$), vector ($O_v=\gamma_\mu$), axial vector ($O_A=\gamma_\mu\gamma_5$) or tensor ($O_T=\sigma_{\mu\nu}$), or some combination of these. Fermi hypothesized that the interaction is vector by analogy to electrodynamics-and to a large extent he was right.

Heisenberg conjectured that perhaps the Fermi interaction was the glue which held nucleons together in the nucleus.[4] Just as there is a connection between the emission of light and the Coulomb interaction between charged particles, he hoped that the virtual exchange of the Fermi fields might account for the stability of the nucleus. A calculation, under the simplifying assumptions that the nuclei can be considered infinitely heavy and that the electron (and neutrino) mass can be neglected,

gives the nuclear potential due to Fermi exchange proportional to $1/r^5$. Since this blows up as $r\to o$, this interaction would result in an unacceptably infinite binding energy. Assuming that for some reason the interaction cuts off at about one fermi (10^{-13}cm), we get good agreement for the binding energy of He^3 and He^4. However, the same cutoff produces a binding for the deuteron which is too weak by a factor of 10^{12}! Because this explanation of nuclear forces is clearly wrong, it inspired Yukawa to propose an alternative: a theory of scalar meson exchange as the source of nuclear binding. Yukawa predicted a new scalar particle of mass $\sim 200\ m_e$; after some confusion, the pion was observed.[5]

Over the next twenty years as more particles were discovered, additional decays were observed which seemed to arise from weak interactions as well. The interactions of the new subatomic particles (if we neglect gravity) seemed to fall into three distinct classes-weak, electromagnetic and strong-each characterized by wildly distinct coupling strengths. The strong interaction, which is responsible for nuclear binding, is characterized by a coupling constant $g^2/4\pi \approx 15$. This is three orders of magnitude larger than the fine structure constant $\alpha=e^2/4\pi$ which characterizes electromagnetic interactions. The weak coupling constant G_β has dimension, but the dimensionless quantity $(G_\beta M_N^2)^2/4\pi$ was generally believed to be a good analog of α for measuring the weak interactions---in the absence of other masses to set the scale for the weak interactions. The weak "coupling constant" $(G_\beta M_N^2)^2/4\pi \cong 10^{-11}$ is nine orders of magnitude smaller than the electromagnetic interaction. If, on the other hand, a considerably larger mass naturally set the scale for the weak interactions, then the weak dimensionless coupling constant might be of the same order as the electromagnetic coupling constant. This is what happens in unified weak-electromagnetic interaction theories.

Among the new weak interaction decays was the decay of the newly found muon: $\mu^-\to e^-+\bar{\nu}_e+\nu_\mu$. Since this is a four fermion decay, it was natural to describe it by an interaction similar to the Fermi theory of β decay

$$H_\mu = G_\mu \bar{\psi}_{\nu\mu}(x) O \psi_\mu(x) \bar{\psi}_e(x) O \psi_{\nu e}(x)$$

with G_μ of the same order as G_β. Similarly the weak decay of the pion $(\pi^- \to \mu^- + \bar{\nu}_\mu)$ could be described by a four fermion interaction, through the sequence $\pi^- \xrightarrow[\text{strong}]{} \bar{p}+n \xrightarrow[\text{weak}]{} \mu^- + \bar{\nu}_\mu$,[6]

$$H_\pi = G_\pi \bar{\psi}_p(x) O \psi_n(x) \bar{\psi}_\mu(x) O \psi_{\nu\mu}(x)$$

Throughout this period, debate raged over what the operators O in these expressions happen to be. Nuclear β decay experiments suggested that the operators involved were either combinations of V and A or combinations of S and T, with the latter favored. μ decay experiments indicated V and A, while pi decay suggested A or P since the pion was found to be pseudoscalar. It appeared that no universal interaction could explain all three decays.

In 1956 an apparently unrelated dilemma associated with the purely hadronic decays of mesons lead to a breakthrough in weak interaction theory. The θ^+ meson which decays into two pions and the τ^+ meson which decays into three pions were found to have the same mass and lifetime. This suggested that they were really the same particle (the kaon); however parity conservation prohibited the same particle from decaying both ways. Lee and Yang solved the dilemma by observing that no experiments to date had verified the invariance of the weak interactions under parity. They conjectured that in fact parity is violated by the weak interactions and suggested that a good experiment to test parity violation was to measure the right-left asymmetry in the β decay of oriented Co^{60}.[7] Their conjecture was verified by C.S.Wu who performed the experiment.[8]

The idea of parity violation opened the way for a universal weak interaction theory, as proposed by Feynman and Gell-Mann in 1958[9] and later modified by Cabibbo.[10] It is based on a V-A current-current interaction

$$H_{weak} = \frac{G}{\sqrt{2}} J_\mu^\dagger J^\mu$$

G is a universal constant. The current J_μ is the sum of a hadronic part and a leptonic part

$$J_\mu^{hadronic} = V_\mu - A_\mu$$

$$J_\mu^{\text{leptonic}} = \bar{\nu}_e \gamma \frac{(1+\gamma_5)}{2} e + \bar{\nu}_\mu \gamma_\mu \frac{(1+\gamma_5)}{2} \mu$$

The weak interactions also describe the weak decays of the so-called strange particles (kaons, lamba, etc); these decays violate the conservation of a new quantum number, strangeness, which is conserved by strong and electromagnetic interactions. To describe strangeness changing decays (such as $\Lambda \rightarrow p + e^- + \bar{\nu}e$) as well as strangeness conserving (such as nuclear β decay) it is commonly assumed that the hadronic current contains two pieces:

$$J_\mu^{\text{hadronic}} = J_\mu^{(\Delta S=o)} \cos\Theta_c + J_\mu^{(\Delta S=1)} \sin\Theta_c$$

θ_c is called the Cabibbo angle and is about 13.7^o.[11] The hadronic currents satisfy well defined algebraic relations among themselves, called current algebra, which enable calculations of some weak hadronic processes even though the exact form of the hadronic current is not specified.

The current-current Lagrangian describes in a universal manner a melange of seemingly disparate phenomena-purely leptonic processes such as μ decay, semileptonic decays of both strange and nonstrange hadrons and even nonleptonic strangeness violating decays. Weak processes, which would ordinarily be masked by the much stronger strong and electromagnetic interactions, can be detected in decays which violate symmetry principles respected by the strong and electromagnetic interactions such as parity conservation, time reversal invariance and strangeness. These tests of the weak interaction Lagrangian are all low energy tests. Another good place for examining the weak interactions is in scattering experiments of leptons, which do not interact strongly. These experiments are hard to come by and are in their early stages with the new neutrino beams.

At the same time that the phenomenological description of the weak interactions evolved, a self-consistent relativistic quantum mechanics was developed, first for quantum electrodynamics and later for more general Lagrangian field theories. In the calculation of cross sections, the new quantum field theories often give meaningless infinite results. How-

ever, by an ingenious process of redefining parameters in the Lagrangian, called renormalization, the calculations are rendered finite. When a theory can be made finite by redefining a finite number of parameters, the theory is said to be renormalizable. Unfortunately, the four-fermi phenomenological theory of the weak interactions, when treated as a field theory, is not renormalizable. At least with our present degree of technical know-how, unrenormalizable field theories do not have predictive power and therefore are not physically interesting. Until the recent advent of unified weak-electromagnetic gauge theories, all attempts to modify the four-fermi theory, so as to reproduce the same phenomenology at low energies but in a dynamically predictable way, failed.

Could we perhaps ignore field theory and treat the phenomenological interaction as a description of the weak interactions at all energies without calculating higher order effects, since it is at higher orders that the infinities crop up? The answer is no, because when we do this we soon run into difficulties with the principle of unitarity.[12] This principle, which states that probability is conserved (namely that the sum of the probability of all possible outcomes of an experiment is 1) is clearly one which we are not willing to give up.

At this impasse many particle physicists took solace in religion, mumbling from Corinthians, "....the weakness of God is stronger than men." However, a possible strategy for constructing a renormalizable weak interaction theory was suggested by Weinberg and Salam in 1967.[13] The technical proof that their strategy actually works was given by 't Hooft in 1971.[14] We are still a good way from the concrete realization of their strategy in a specific model which elegantly describes the physics of the weak interactions.

II. Plan of the Thesis

In the first part of the introduction we have given a brief history of weak interaction theory up until the impasse which led to the development of the unified weak-electromagnetic gauge theory strategy. In Chapter 2, we

describe the basic ideas underlying the new gauge strategy. We start with a Lagrangian, invariant under a local gauge group, containing massless vector mesons. However the gauge symmetry of the Lagrangian is not manifest in the states of the system; the symmetry is said to be spontaneously broken. In the gauge theories, we break the symmetry by giving nonzero vacuum expectation value to some scalar fields, called Higgs fields. Generally when a symmetry of the Lagrangian is spontaneously broken, the theory contains zero mass scalar particles called Goldstone bosons. However when a local gauge theory is spontaneously broken, the Goldstone bosons which are linear combinations of the Higgs fields do not appear as physical particles. Instead they become the longitudinal components of the massless vector mesons, which now acquire mass. It is these massive vector mesons which transmit the weak interaction force between pairs of fermions, just as the exchange of photons is responsible for the electromagnetic interaction between charged particles. The theory which results can replicate the low energy phenomenology of the Fermi theory and it is renormalizable as well. To illustrate how these ideas can be concretized in a specific model, we begin in Chapter 2 by describing the Weinberg SU(2)xU(1) model of leptons.[13] This was the pioneering work in the field; since then a plethora of models have been built, none of them entirely compelling. But we have learned a great deal in working them out and thinking about their consequences and pitfalls. In the concluding section of this chapter, we discuss some of the exciting prospects for the new gauge strategy and the perplexing problems which have yet to be solved.

In Chapter 3 and 4 we make a small contribution towards solving some of the problems which arise in applying the gauge strategy. We focus on the role of the Higgs scalars in the spontaneous breakdown of the theory. We find that a lot of physics may lurk in the Higgs sector.

In Chapter 3 we ask ourselves the following question: how can we break the symmetry in a unified weak-electromagnetic gauge theory in such a way that all of the weak vector mesons acquire mass but that the photon remains massless? We find that all but a few low-dimensional choices of Higgs representations can break the symmetry in the necessary way. Given the Higgs

representation, we then examine what restrictions are placed on the electric charge operator of the theory. We work out in detail the analysis for a few specific symmetry groups. We also give all the possible Higgs representations, which can break the symmetry down to electromagnetism, for all the unexceptional classical Lie groups.

In Chapter 4 we study the effects on calculable quantities of varying the Higgs content in a specific model. The model is not realistic because it does not include strange particles, but we find it an interesting testing ground nonetheless.

One quantity, which most physicists agree should be determined in a complete theory, is the proton-neutron mass difference. In this model, it is calculable. The question of the proton-neutron mass difference has haunted particle physicists for decades. Previous calculations based solely on electromagnetism resulted in either an infinite mass difference or else the wrong sign. However, the discovery of gauge theories provided the possibility that both the weak and electromagnetic interactions could conspire together to produce a neutron heavier than the proton. In a spontaneously broken gauge theory, if a mass difference or mass is zero in zeroth order, without any restrictions on the parameters of the Lagrangian, even after the symmetry is broken, then that quantity is necessarily finite and calculable.

In the context of our model we find that even though a mass or mass difference may be calculable, its value depends critically on the symmetry breaking mechanism. For example, we find that the proton-neutron mass difference is a function of the way in which the symmetry is broken; the mass difference can be either positive or negative depending on the way the Higgs content is chosen. Since this model is preliminary (as are all existing models) no definitive value of the mass difference is obtained. Although a quantity may be calculable, namely a finite function of the renormalized parameters of the theory, it is quite another matter for it to be computable—to know what that function is. Until we understand how the strong interactions fit into the weak-electromagnetic framework, even though the mass difference is still calculable if the strong interaction

theory is renormalizable, it is not really computible since its value will be modified by the presence of the strong interactions. However, there is a class of strong interaction theories, called asymptotically free, which act like free field theories at high momenta. There are experimental indications based on electroproduction data, that the strong interactions are in fact asymptotically free. For these theories, the mass difference calculation to lowest order is unaffected by the presence of the strong interactions at least in a naive quark model.[15]

In the same model, we examine a possible mechanism for incorporating pions into weak-electromagnetic gauge theories in the Higgs sector. One of the goals of particle physics is to understand why pions are so light compared to other strongly interacting particles. In our model we have a mass degenerate triplet in the Higgs sector which interacts strongly with nucleons. We identify this triplet with the pion triplet. The pion mass difference, due to weak and electromagnetic effects, is then calculable. If we impose an extra symmetry on the theory, we find that the pions are massless in zeroth order, but pick up mass, which is calculable, in higher orders. This model is the first implementation in a weak-electromagnetic theory of a general mechanism suggested by Weinberg for generating scalar particles of very small calculable mass, called pseudoGoldstone bosons. Our calculation illustrates some of the problems and prospects of implementing this idea in a more realistic model.

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2. A NEW STRATEGY: UNIFIED WEAK-ELECTROMAGNETIC GAUGE THEORIES

I. Spontaneous Symmetry Breaking

In both classical and quantum field theory, the symmetries of the theory correspond to invariances of the Lagrangian. In the conventional realization of the symmetry, the ground state or state of lowest energy (called the vacuum in quantum field theory) is invariant under the transformations of the symmetry group and particles belong to mass degenerate representations of the group. However, there is no reason to expect that the ground state of a system necessarily embodies the invariances of the Lagrangian. Although the set of equations which determine the dynamics of a system is invariant since it comes from the Lagrangian, the solutions need not be if the boundary conditions are not also invariant.

In classical physics, for example, consider an infinite 2-dimensional array of magnetic dipoles which interact ferromagnetically through nearest neighbor interactions. In the ground state all the dipoles are aligned parallel to one another. There are infinitely many possible ground states - one for each vector in the plane. Although the interaction is invariant under rotations in the plane, the physical ground state is not - it singles out a unique direction, the direction in which all the magnets point.

In quantum field theory, if the Lagrangian is invariant under a symmetry but the vacuum is not, then the states of the system are not covariant with respect to rotations of the group. A symmetry manifest in the Lagrangian but not in the ground state is said to be spontaneously broken. A theorem of field theory states that if the symmetry is not manifest in the ground state, then it is not manifest in the states. (This terminology is somewhat misleading since the symmetry is not broken - it is merely hidden.) In that case, if the "broken" symmetry is continuous and if the theory obeys the usual axioms of quantum field theory (Lorentz covariance, positive definite Hilbert space, locality,...) then the symmetry is realized by the appearance of zero mass spinless particles, called Goldstone bosons - one for each generator of the group.[1]

As an example of the two possible realizations of a Lagrangian symmetry, we consider a simple model of scalar fields:

$$\mathcal{L}_I = -\tfrac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2] - B^2(\sigma^2 + \vec{\pi}^2 - A)^2 \qquad (1)$$

This Lagrangian is invariant under the continuous group $SU(2)_L \times SU(2)_R$ with the $\sigma, \vec{\pi}$ fields assigned to the representation

$$H = \begin{pmatrix} \frac{\sigma + i\pi_o}{\sqrt{2}} & i\pi^+ \\ i\pi^- & \frac{\sigma - i\pi_o}{\sqrt{2}} \end{pmatrix} \qquad T_L = 1/2, \; T_R = 1/2$$

[This is just the σ-model of Gell-Mann and Levy without nucleons.][2] Classically, the ground state of the system is the state which minimizes the energy density H

$$H = \tfrac{1}{2}[(\partial_o\sigma)^2 + (\partial_o\vec{\pi})^2] + \tfrac{1}{2}[(\vec{\nabla}\sigma)^2 + (\vec{\nabla}\vec{\pi})^2] + V(\sigma,\vec{\pi})$$

where the potential V is $B^2(\sigma^2+\vec{\pi}^2-A)^2$. Clearly the minimum is achieved for $\langle\sigma\rangle$, $\langle\vec{\pi}\rangle$ = constant at the minimum of the potential, given by

$$\left(\frac{\partial V}{\partial\phi_i}\right)_{VEV} = 0 \qquad \left(\left|\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right|\right)_{VEV} \geqslant 0$$

where ϕ_i are the independent scalar fields. This stability condition for the vacuum carries over into the quantum field theory.

The potential looks quite different in the two cases $A<0$ and $A>0$ (see Figure 1). In case $A<0$, the minimum is achieved for $\langle\sigma\rangle=0$, $\langle\vec{\pi}\rangle=0$ and we can therefore do conventional perturbation theory. In case $A>0$, the potential minimum occurs at $\langle\sigma^2+\vec{\pi}^2\rangle=A$. Since the calculational apparatus of perturbation theory in quantum field theory is based upon the assumption that fields have zero vacuum expectation value, we must reexpress the Lagrangian in terms of new fields which do not have vacuum expectation value. There are an infinite number of ways in which we can choose new fields, corresponding to an infinite degeneracy of the vacuum. Any choice is physically equivalent; therefore we may take $\langle\sigma\rangle=\sqrt{A}$, $\langle\vec{\pi}\rangle=0$, and define a new field

$$\sigma' = \sigma - \sqrt{A}\,, \quad \langle\sigma'\rangle = 0$$

Rewriting the Lagrangian in terms of the new field, we find

$$\mathcal{L}_{II} = -\tfrac{1}{2}[(\partial_\mu\sigma')^2 + (\partial_\mu\vec{\pi})^2] - B^2(\sigma'^2 + \vec{\pi}^2 + 2\sqrt{A}\,\sigma')^2 \qquad (2)$$

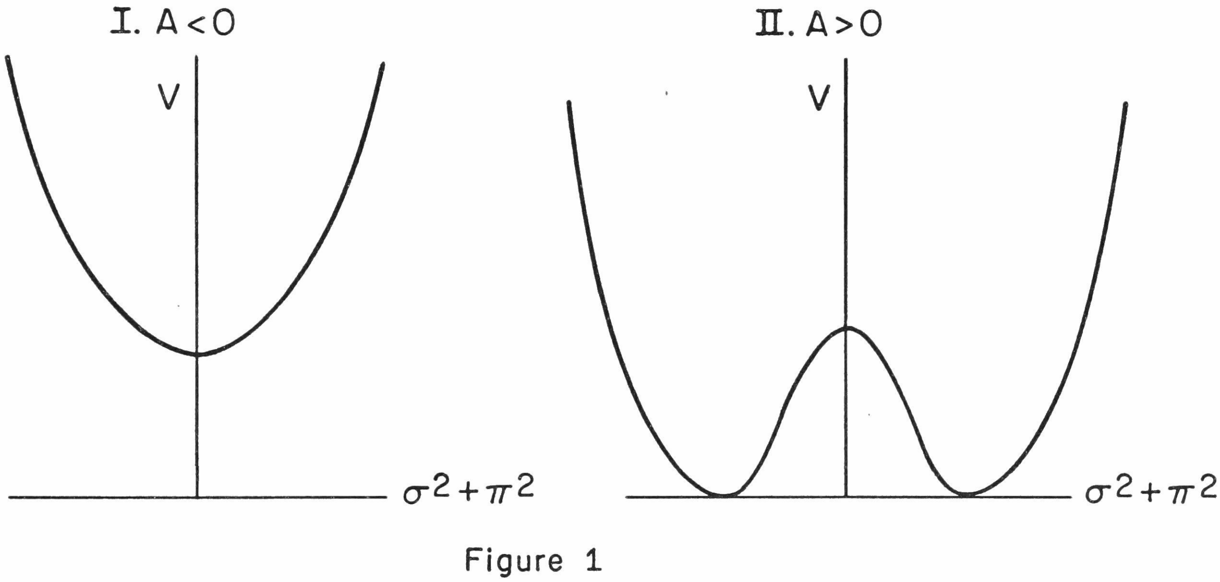

Figure 1. Shape of the potential in the two cases I. A<0 and II. A>0.

L_I, which is the Lagrangian appropriate to $A<0$, describes a theory in which the symmetry is conventionally realized -- the σ and $\vec{\pi}$ fields are a mass degenerate multiplet with mass $m^2= -4AB^2$. When the symmetry is spontaneously broken for $A>0$, the $(\sigma,\vec{\pi})$ multiplet splits into a singlet σ with mass $m^2= 8AB^2$ and a massless $\vec{\pi}$ triplet. The Lagrangian is now invariant under SU(2) instead of SU(2)xSU(2). For each generator of the spontaneously broken SU(2) group, the theory develops a massless scalar particle - $\pi^+ \; \pi^o \; \pi^-$.[3]

The appearance of zero mass spinless particles in spontaneously broken quantum field theories is a general phenomenon. Goldstone showed that in a manifestly causal and Lorentz covariant field theory, if the Lagrangian is invariant under a continuous symmetry group, then either the vacuum state is invariant under the transformation or there must be massless spinless particles in the theory. These massless bosons may be elementary fields in the Lagrangian, as in the σ-model, or they may be bound states of fermions. The appearance of Goldstone bosons as bound states occurs in the four fermion interaction model of Nambu and Jona-Lasinio.[1] Since there are no known massless scalar particles, the idea of using spontaneously broken field theories in particle physics to describe nature was not considered seriously, except as a possible approximation for the smallness of the pion mass.

We outline a nonrigorous proof of the Goldstone theorem based on Goldstone, Salam and Weinberg.[4] A more rigorous version has been given by Streater.[5] Suppose the Lagrangian is invariant with respect to a symmetry group, under which the fields transform as

$$\delta^a \phi_i = \varepsilon T^a_{ij} \phi_j \tag{3}$$

Then the currents $J^a_\mu = i\left(\frac{\delta \mathcal{L}}{\delta \partial^\mu \phi_i}\right) \; \delta^a \phi_i$ are conserved, whether or not the vacuum is invariant under the transformation. From the canonical commutation relations, $Q^a = \int J^a_o d^3x$ is the generator of the transformation

$$[Q^a, \phi_i] = T^a_{ij} \phi_j \tag{4}$$

Suppose that the vacuum is not invariant under the transformation. Then $Q^a|0> \neq 0$. In particular we assume that there exists a set of spinless fields, which need not be fundamental, transforming as

$$[Q^a, \phi_i] = T^a_{ij} \phi_j$$

If the vacuum is not invariant, then

$$<0|[Q^a,\phi_i]|0> = T^a_{ij} <\phi_j> \neq 0 \tag{5}$$

for some ϕ_i. [If $<\phi>$ is nonzero, ϕ cannot carry spin since angular momentum conservation is an exact symmetry, nor can it carry charge since electric charge symmetry is exact.] Writing the spectral representation for the commutator we find,

$$<0|[J_\mu{}^a(x),\phi_i(o)]|0> = \partial_\mu \int dm^2 \Delta(x,m^2)\rho_i^a(m^2) \tag{6}$$

where Δ is the usual causal Green's function for mass m and

$$p^\mu \Theta(p^o)\rho_i^a(-p^2) = -(2\pi)^3 \sum_n \delta(p-p^n)<o|J_\mu^a(o)|n><n|\phi_i(o)|o> \tag{7}$$

The ability to express the commutator in this form depends on the manifest Lorentz covariance of the theory. Since the current is conserved,

$$0 = <o|[\partial^\mu J_\mu^a(x),\phi_i(o)]|o> = \Box^2 \int dm^2 \Delta(x,m^2)\rho_i^a(m^2) = \int dm^2 \Delta(x,m^2) m^2 \rho_i^a(m^2)$$

Hence $m^2\rho_i^a(m^2)=0$ which implies

$$\rho_i{}^a(m^2) = N_i{}^a\delta(m^2)$$

$$<o|[J_\mu^a(x),\phi_i(o)]|o> = N_i^a \partial_\mu D(x) \tag{8}$$

where $D(x)= \Delta(x,o)$. Generally, we would expect N_i^a to be zero and hence there would be no massless particles in the theory. However, in our case this is impossible. From equation (5), we know

$$0 \neq <o|[Q^a,\phi_i]|o> = \int d^3x <o|[J_o^a(x),\phi_i(o)]|o> = N_i{}^a \int d^3x \partial_o D(x) = N_i^a$$

Thus the spectral representation (7) must include states of zero mass.

We can arrive at a physical understanding of the appearance of zero mass scalar particles from the degeneracy of the vacuum. If the Lagrangian is invariant under a continuous symmetry group but the vacuum is not, then there are an infinite number of states which have the same energy as the vacuum and which are obtained from one another by infinitesimal transformations under the group. Physically, these extra vacua are obtained by adding zero-velocity massless scalars to the particular state which we pick as the physical vacuum.

II. Evasion of the Goldstone Theorem: The Higgs Mechanism

The proof of the Goldstone theorem depends critically on the manifest Lorentz covariance of the theory. There is, however, a class of quantum field theories which cannot be quantized in a manifestly Lorentz covariant fashion if we insist that the Hilbert space in which we operate has a positive definite norm.[6] These theories, called gauge theories, are invariant under local symmetry transformations. For each independent symmetry generator, there is a massless vector meson in the theory. Quantum electrodynamics is a well known illustration of an abelian U(1) gauge invariant theory; the massless vector meson is, of course, the photon. However, aside from the photon, there are no other massless vector mesons in nature; this appears to limit the usefulness of gauge theories for physics.

The simplest example of a gauge theory is electrodynamics. If one insists on the positive definiteness of the inner product in Hilbert space, then the canonical quantization procedure runs into difficulties. Although the electromagnetic field vector A_μ has four independent components, the photon has only two independent polarizations. Although one can choose three independent polarizations (as for a massive vector meson field) in a Lorentz covariant manner by imposing the subsidiary condition $\varepsilon_\mu k^\mu=0$, we must impose a further noncovariant condition to eliminate the extra degree of freedom. For example, we can choose $\varepsilon_\mu \eta^\mu=0$ where $\eta^\mu=(1,0,0,0)$ is a pure timelike vector. In that case the two independent polarization vectors are spacelike and orthogonal to the direction of propagation of the field. Introducing a "free" vector η_μ into the theory breaks the manifest covariance.

The Lagrangian for the electrodynamics of a set of scalar fields ϕ_i with charge Q_i and spinor fields ψ_n with charge Q_n is the form:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}(D_\mu\phi_i)^*(D^\mu\phi_i) - \bar{\psi}\gamma^\mu D_\mu\psi - \bar{\psi}m_o\psi - P(\phi) - \bar{\psi}\Gamma_i\psi\phi_i \tag{9}$$

where

a) $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

b) The gauge covariant derivatives of the scalar fields ϕ_i are

$$D_\mu\phi_i = \partial_\mu\phi_i + iQ_i\phi_i A_\mu$$

c) The gauge covariant derivatives of the spin 1/2 fields ψ_n are

$$D_\mu\psi_n = \partial_\mu\psi_n + iQ_n\psi_n A_\mu$$

d) $P(\phi)$ is a fourth order polynomial in the scalar fields and Γ_i are Yukawa couplings. Both $P(\phi)$ and Γ_i couplings conserve charge. (The reason that $P(\phi)$ contains at most fourth order terms is that otherwise the theory is not renormalizable.)

The Lagrangian (9) is invariant under the local transformations

$$\begin{aligned} A_\mu(x) &\to A_\mu(x) - \partial_\mu\Lambda(x) \\ \phi_i(x) &\to \phi_i(x) + iQ_i\Lambda(x)\phi_i(x) \\ \psi_n(x) &\to \psi_n(x) + iQ_n\Lambda(x)\psi_n(x) \end{aligned} \tag{10}$$

The fact that the transformation can vary from point to point in space-time as $\Lambda(x)$ is what makes the theory locally gauge invariant. (This is often called gauge invariance of the second kind; if the theory is invariant only for Λ=constant it is said to be gauge invariant of the first kind.) This is the most general Lagrangian that is locally invariant under the simple one-parameter abelian gauge group U(1).

To see what happens when a gauge theory is spontaneously broken, we consider the simple version of the abelian gauge Lagrangian discussed by Higgs.[7] The Lagrangian is constructed from a gauge field A_μ and a complex scalar field ϕ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) \tag{11}$$

where

$$D_\mu\phi = \partial_\mu\phi + iQ\phi A_\mu$$

$$V(\phi^*\phi) = a\phi^*\phi + b(\phi^*\phi)^2$$

Suppose we allow the symmetry to be spontaneously broken by choosing $a<o$: $\langle\phi\rangle=v$, v real. The condition that the potential be a minimum is then

$$\left\langle\frac{\partial V}{\partial\phi}\right\rangle = \langle a\phi^* + 2b(\phi^*\phi)\phi^*\rangle = 0$$

$$v(a+2bv^2) = 0, \quad v^2 = -a/2b \tag{12}$$

We rewrite ϕ as $\phi=v+\phi_1+i\phi_2$ where $\langle\phi_1\rangle=\langle\phi_2\rangle=0$ and ϕ_1,ϕ_2 are real. Substituting

in the Lagrangian and using condition (12), we find

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}(\partial_\mu\phi_1 + QA_\mu\phi_2)^2 - \tfrac{1}{2}(\partial_\mu\phi_2 - QA_\mu\phi_1)^2 - \tfrac{1}{2}Q^2v^2A_\mu A^\mu$$
$$+QvA_\mu(QA^\mu\phi_1 - \partial^\mu\phi_2) - b(\phi_1^2+\phi_2^2)^2 - 4vb\phi_1(\phi_1^2+\phi_2^2) - (a+6bv^2)\phi_1^2$$

The gauge meson picks up mass $\mu^2 = Q^2v^2$, ϕ_1 picks up mass $m^2 = 2(a+6bv^2) = 8bv^2$ and ϕ_2 is massless. At first glance, it appears that ϕ_2 is a physical Goldstone boson for the theory. What really happens, however, is that ϕ_2 decouples from the rest of the particles and disappears from the physical spectrum. The extra degree of freedom associated with ϕ_2 becomes the longitudinal component of the now massive vector meson. To see this, we rewrite the Lagrangian in terms of new fields:

$$\phi = (v+\rho)e^{i\theta}$$

$$A_\mu = B_\mu - \frac{1}{Q}\partial_\mu\theta$$

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \tfrac{1}{2}(\partial_\mu\rho - iQ\rho B_\mu)(\partial^\mu\rho + iQ\rho B^\mu) - \tfrac{1}{2}Q^2v^2B_\mu B^\mu -$$
$$b\rho^4 - 4bv\rho^3 - 4bv^2\rho^2$$

As predicted the Θ degree of freedom no longer appears explicitly in the Lagrangian - instead it is incorporated into the massive B_μ meson field.

This phenomenon is more general and happens whenever a gauge theory is spontaneously broken. The would-be Goldstone bosons of the spontaneously broken gauge theory, called Higgs mesons, get "eaten-up" by the massless vector mesons, which acquire mass. The extra degree of freedom associated with massive vector mesons (3 instead of 2) is supplied by the Higgs scalar, which disappears from the particle spectrum. No massless bosons need survive. Thus the spontaneous symmetry breaking strategy and the gauge field program stand or fall together-each saves the other from its zero mass problem.

III. Non-Abelian Gauge Theories

We would like to be able to use spontaneously broken gauge theories to describe the weak interactions. Since the weak interactions are vectorial in character and since we now know how to make the gauge vector mesons massive via spontaneous symmetry breaking, we seem to be well on our way. Furthermore, since electrodynamics is also described by a gauge theory, it seems like a good idea to try to combine the weak interactions and electrodynamics into a single unified theory. In breaking the symmetry of the unified theory, we must break all the symmetries except for electric charge symmetry such that all the vector mesons except for the photon acquire mass. (see Chapter 3) Another justification we have for wanting to unify weak and electromagnetic interactions, aside from the elegance of such a simplification, is a basic principle of current algebra called CVC (conserved vector current hypothesis). It states that the strangeness conserving part of the weak vector current and the isovector part of the electromagnetic current are members of the same isotopic triplet. This suggests an intimate connection between the two types of interactions. In addition, the theory of vector meson electrodynamics is not generally renormalizable--in unifying the two theories we get a consistent renormalizable electrodynamics for the weak fields as well.

Gell-Mann and Glashow have shown that all locally invariant field theories are gauge theories associated with a Lie group.[8] They are generalizations of electrodynamics which is associated with the simplest Lie group, the one-parameter group U(1). Given a gauge group G, the Lagrangian is constructed from a set of massless vector meson fields A^α_μ (one for each independent generator of the group), a multiplet of scalar fields assigned to a representation (perhaps reducible) of the group and a multiplet of fermion fields. Suppose that the group generators satisfy the commutation relations

$$[Q^\alpha, Q^\beta] = iC^{\alpha\beta\gamma}Q^\gamma$$

In analogy with electrodynamics, the most general Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}F^\alpha_{\mu\nu}F^{\mu\nu\alpha} - \tfrac{1}{2}(D_\mu\phi_i)^*(D^\mu\phi_i) - \bar{\psi}\gamma^\mu D_\mu\psi - \bar{\psi}m_o\psi - P(\phi) - \bar{\psi}\Gamma_i\psi\phi_i \tag{13}$$

where a) The gauge covariant curl is $\partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu - C^{\alpha\beta\gamma}A_\mu{}^\beta A_\nu{}^\gamma$

b) The gauge covariant derivative of the scalar field is

$$D_\mu \phi_i = \partial_\mu \phi_i + i(\theta_\alpha)_{ij}\phi_j A^\alpha_\mu$$

where Θ_α is the representation matrix for the scalar multiplet.

c) The gauge covariant derivative of the spinor field is

$$D_\mu \psi_n = \partial_\mu \psi_n + i(t_\alpha)_{nm}\psi_m A^\alpha_\mu$$

where t_α is the representation matrix for the fermion multiplet.

d) The fermion mass matrix is G invariant, $P(\phi)$ is a fourth order G invariant polynomial and the Yukawa coupling is G invariant:

$$[t_\alpha, \gamma_o m_o] = 0$$

$$\frac{\partial P}{\partial \phi_i}(\theta_\alpha)_{ij}\phi_j = 0$$

$$[t_\alpha, \gamma_o \Gamma_i] = -(\theta_\alpha)_{ij}\gamma_o \Gamma_j$$

This Lagrangian is the most general one invariant under the local transformations

$$A^\alpha_\mu(x) \rightarrow A^\alpha_\mu(x) - \partial_\mu \Lambda^\alpha(x) + C^{\alpha\beta\gamma}\Lambda^\beta A^\gamma_\mu(x)$$

$$\phi_i(x) \rightarrow \phi_i(x) + i\theta^\alpha_{ij}\Lambda^\alpha \phi_j(x)$$

$$\psi_n(x) \rightarrow \psi_n(x) + it^\alpha_{nm}\Lambda^\alpha \psi_m(x)$$

The canonical quantization of non-abelian gauge theories is no easy task. Just as in electrodynamics, there are redundant variables which do not correspond to independent degrees of freedom. But the non-abelian case is much more complicated. Even when there are no scalars or fermions present, the Lagrangian is an interacting one, unlike in the abelian case. In a non-abelian theory, the gauge fields carry nontrivial quantum mumbers of the group. Since a gauge field couples to everything with nonzero quantum number associated with it, just as the photon couples to everything with electric charge, the non-abelian gauge fields are necessarily self interacting.

A way of calculating non-abelian gauge theories was developed by Fadeev and Popov using the path integral formalism.[9] Their method involves integrating out the extra gauge degree of freedom from the generating function for the Green's functions of the theory. However, the calculations are formal in the

sense that they involve the cavalier manipulation of functional integrals which are not necessarily well defined. Moreover, in developing the Feynman rules it is in most gauges necessary to introduce (as a calculational device) scalar ghost fields which obey Fermi statistics. This calls into question the unitarity of the theory.

However, the validity of the path integral approach was subsequently verified by the canonical quantization of the theory.[10] In the axial gauge (also known as the Arnowitt-Fickler gauge[11]), defined by the gauge condition $A_3^a=0$, the quantization procedure is relatively straight forward and does not involve any ghost particles. The resulting Feynman rules agree with those derived in this gauge via the path integral method and the theory is manifestly unitary.[12]

When the non-abelian gauge theory of eqtn (13) is spontaneously broken as the scalar fields develop nonzero vacuum expectation value ($\langle\phi_i\rangle=\lambda_i$), the new theory is defined in terms of shifted fields which have zero vacuum expectation value

$$\phi_i{}' = \phi_i - \lambda_i$$

Because of the symmetry breaking, the gauge vector mesons which correspond to broken symmetry generators acquire nonzero mass. This comes from the $-1/2(D_\mu\phi_i)^*(D^\mu\phi_i)$ term in the Lagrangian. The zeroth order mass matrix becomes

$$\mu^2_{\alpha\beta} = -(\theta_\alpha\lambda)_i(\theta_\beta\lambda)_i \tag{14}$$

The scalar and fermion mass matrices are also shifted:

$$M^2_{ij} = \left\langle \frac{\partial^2 P(\phi)}{\partial\phi_i\partial\phi_j} \right\rangle$$

$$m = m_o + \Gamma_i\lambda_i \tag{15}$$

The Feynman rules, in a convenient class of gauges parameterized by the quantity ξ, are given by an effective interaction Lagrangian:

$$\begin{aligned}\mathcal{L}' = &- \tfrac{1}{2}(\partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu)C^{\alpha\beta\gamma}A^{\beta\mu}A^{\gamma\nu} - \tfrac{1}{4}C^{\alpha\beta\gamma}C^{\alpha\delta\varepsilon}A^\beta_\mu A^\gamma_\nu A^{\delta\mu}A^{\varepsilon\nu}\\ &-i\partial_\mu\phi_i'(\theta_\alpha)_{ij}\phi_j'A^{\alpha\mu} - (\theta_\beta\phi_\alpha\lambda)_i\phi_i'A^\beta_\mu A^{\beta\mu} - \tfrac{1}{2}(\theta_\beta\theta_\alpha)_{ij}\phi_i'\phi_j'A^\alpha_\mu A^{\beta\mu}\\ &-i\bar\psi\gamma^\mu t_\alpha\psi A_{\alpha\mu} - \tfrac{1}{6}f_{ijk}\phi_i'\phi_j'\phi_k' - \tfrac{1}{24}f_{ijkl}\phi_i'\phi_j'\phi_k'\phi_l' - \bar\psi\Gamma_i\psi\phi_i\\ &-\partial_\mu\omega^*_\alpha C^{\alpha\beta\gamma}\omega_\beta A^{\gamma\mu} - \xi^{-1}\omega_\alpha^*\omega_\beta(\theta_\beta\theta_\alpha\lambda)_i\phi_i'\end{aligned} \tag{16}$$

where ω_α are the ghost fields and $f_{ijk} = \left\langle \frac{\partial^3 P(\phi)}{\partial\phi_i \partial\phi_j \partial\phi_k} \right\rangle$, $f_{ijkl} = \left\langle \frac{\partial^4 P(\phi)}{\phi_i \phi_j \phi_k \phi_l} \right\rangle$.

The field propagators in the ξ gauge are

$$\Delta^A_{\alpha\mu,\beta\nu}(k) = g_{\mu\nu}(k^2+\mu^2)^{-1}_{\alpha\beta} + (1-\xi)k_\mu k_\nu[(k^2+\mu^2)^{-1}(\xi k^2+\mu^2)^{-1}]_{\alpha\beta}$$

$$\Delta^\phi_{ij}(k) = (k^2+M^2)^{-1}_{ij} + (\theta_\alpha\lambda)_i(\theta_\beta\lambda)_j(k^2)^{-1}(\xi k^2+\mu^2)^{-1}_{\alpha\beta}$$

$$\Delta^\psi_{ij}(k) = (i\not{k}+m)^{-1}_{ij}$$

$$\Delta^\omega_{\alpha\beta}(k) = \xi(\xi k^2+\mu^2)^{-1}_{\alpha\beta}$$

The derivation of these rules is beyond the scope of this work. Of course, the physics is independent of the gauge ξ.[13]

For a theory to be physically interesting, we must be able to redefine the parameters in the Lagrangian so that all but a finite and preferably small number of the scattering cross sections and decay rates are calculable. When a theory has this property it is said to be renormalizable. The phenomenological Fermi theory of the weak interactions, when considered as a field theory, is not renormalizable and therefore unsatisfactory. We know that the abelian gauge theory, electrodynamics, is renormalizable. Furthermore, the massive abelian vector meson theory is renormalizable when the vector meson is coupled to a conserved current. On the other hand, Boulware showed that the conventional massive non-abelian vector meson theory (where the vector meson mass terms are put in by hand in the Lagrangian) is never renormalizable--even if the currents are conserved.[14] Therefore, attempts to describe the weak interactions by a conventional intermediate vector meson theory were not successful

However, it can be shown that the non-abelian gauge theory is renormalizable. The original proof is due to 't Hooft - it is very complicated and will not be given here. When a non-abelian gauge theory is spontaneously broken, the renormalizability argument can be extended to the broken theory.[15] Thus we have the exciting new possiblity of a renormalizable massive vector meson theory for the weak interactions.

IV. An Example: The Weinberg SU(2)xU(1) Model

In the past few years there has been a proliferation of models based on various gauge groups and containing different numbers of as yet undetected particles. Each has the virtue of explaining some aspect of weak interaction phenomenology, but none is entirely compelling. As an example of how gauge theories might be used to describe a unified theory of the weak and electromagnetic interactions, we will examine the first model to do this - the SU(2)xU(1) model of Weinberg.[16] The version we describe contains only leptons, although it can be extended to include hadrons as well.

The Weinberg lepton model is based on the gauge group SU(2)xU(1). The leptons are assigned to group multiplets

$$L = \tfrac{1}{2}(1+\gamma_5)\binom{\nu_e}{e^-} \qquad\qquad T = 1/2,\ Y = -1/2$$

$$R = \tfrac{1}{2}(1-\gamma_5)\ e^- \qquad\qquad T = 0,\ Y = -1$$

The electric charge operator is $Q=T_3+Y$. The Higgs scalars introduced to break the symmetry are

$$\phi = \begin{pmatrix}\phi^+\\ \phi^o\end{pmatrix}, \quad \langle\phi^o\rangle = \lambda\ (\lambda \text{ real}),\ T = 1/2,\ Y = 1/2$$

To complete the picture, we have a triplet $\vec{A}_\mu$ and a singlet B_μ of gauge fields. The most general renormalizable gauge invariant Lagrangian constructable from these fields is

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu x\vec{A}_\nu)^2 - \tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2-\bar{R}\gamma^\mu(\partial_\mu-ig'B_\mu)R-\bar{L}\gamma^\mu(\partial_\mu+ ig\tfrac{\vec{\tau}}{2}\cdot\vec{A}_\mu-i\tfrac{g'}{2}B_\mu)L - \tfrac{1}{2}|\partial_\mu\phi+ig\tfrac{\vec{\tau}}{2}\cdot\vec{A}_\mu^\phi+i\tfrac{g'}{2}B_\mu\phi|^2-G_e(\bar{L}\phi R+\bar{R}\phi\ L)-a\phi^\dagger\phi-b(\phi^\dagger\phi)^2 \qquad (17)$$

When we shift fields (define $\phi_1 = \dfrac{\phi_o+\phi_o^\dagger-2\lambda}{\sqrt{2}}$, $\phi_2 = \dfrac{\phi_o-\phi_o^\dagger}{\sqrt{2}i}$), the first four terms of the Lagrangian remain unchanged, whereas the last four contain scalar interactions with vector mesons, fermions and self-interactions as well as the new mass terms

$$-\tfrac{1}{8}\lambda^2 g^2({A_\mu^1}^2+{A_\mu^2}^2)-\tfrac{1}{8}\lambda^2(gA_\mu^3+g'B_\mu)^2-\lambda G_e\bar{e}e+a{\phi_1}^2$$

Thus the charged vector mesons acquire mass $\mu_W^2 = 1/4\lambda^2 g^2$, a neutral vector meson (which we call the Z meson) has mass $\mu_Z^2 = 1/4\lambda(g^2+g'^2)$ and the massless photon is the linear combination

$$A_\mu = \frac{-g' A_\mu^3 + g B_\mu}{\sqrt{g^2+g'^2}}$$

The electric charge is $e = gg'/\sqrt{g^2+g'^2}$. The electron, which started out massless, picks up mass λG_e and all of the scalars except for ϕ_1 disappear from the spectrum. The coupling of the vector mesons with the leptons, expressed in terms of the vector meson eigenstates, is

$$\frac{ig}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu_e W_\mu + \frac{igg'}{\sqrt{g^2+g'^2}}\bar{e}\gamma^\mu e A_\mu + i\sqrt{\frac{g^2+g'^2}{4}}\left[\frac{(3g'^2-g^2)}{g'^2+g^2}\bar{e}\gamma^\mu e - \bar{e}\gamma^\mu\gamma_5 e + \bar{\nu}\gamma^\mu(1+\gamma_5)\nu\right]Z_\mu$$

If we calculate μ decay in the Weinberg model and compare it with the four-fermion interaction (see Figure 2), they coincide for $q^2 \ll \mu_W^2$ provided that

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\mu_W^2} = \frac{1}{2\lambda^2}$$

Since g (and g') are necessarily larger than e $\left(\frac{g}{e} = \sqrt{1+\frac{g^2}{g'^2}} > 1\right)$, we arrive at a lower bound on the charged vector meson mass:

$$\mu_W^2 = \frac{\sqrt{2}\, g^2}{8G_F} > \frac{\sqrt{2}\, e^2}{8G_F} \approx 1600\ m^2_{proton}$$

Therefore, the W meson must be heavier than 40 GeV -- and the Z meson is even heavier. The appearance of vector mesons of this order of magnitude is common to all gauge models.

The existence of neutral massive vector interactions is a new feature of almost all gauge theories (except for the O(3) model of Georgi and Glashow[17]). Although it is possible to put neutral currents in by hand in the Cabibbo theory, they were generally thought to be absent. Experimental tests for neutral currents and their properties are important for determining the validity of the gauge approach. The detection of very massive vector mesons and/or heavy leptons will also be crucial tests, but will require very high energy leptonic beams.

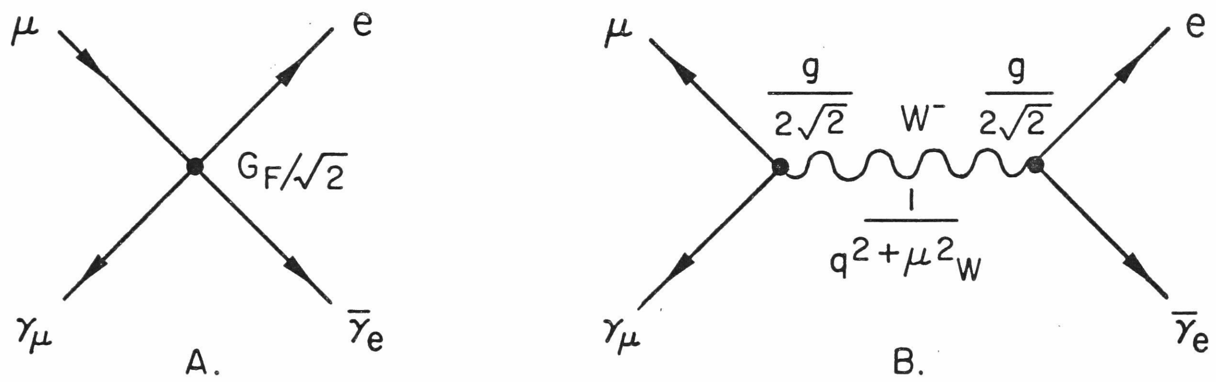

Figure 2

Figure 2. μ decay A. Four fermion interaction, B. Weinberg SU(2)xU(1) model interaction.

V. Problems and Prospects for the New Gauge Theories

The idea of using spontaneously broken gauge theories to describe the weak interactions has opened the way for solving several outstanding problems in particle physics. Perhaps most importantly, we have the means to construct a renormalizable theory of the weak interactions which incorporates the successes of the phenomenological theory at low energies. Equally exciting is the ability to explain two seemingly disparate phenomena-the electromagnetic and weak interactions-by a single unified field theory. Some think that the strong interactions can also be explained by gauge theories, although there remains the calculational impasse of the large coupling constant.

One of the most promising features of gauge theories is the possibility of calculating masses and mass differences. Previously in renormalizable field theories, if a bare mass or mass difference vanished, then either it remained zero to all orders because of an underlying symmetry of the Lagrangian or else it was infinite. In a spontaneously broken gauge theory, if the zeroth order relation holds in the presence of all coupling constants not subject to artificial constraints, then there are no possible counterterms to cancel infinities in the masses or mass differences. Then all higher corrections are finite since the theory is renormalizable (see Chapter 4 for more details).

Although there are a variety of gauge theory models which reproduce in a more or less phenomenological way the known results for low energies, they predict new phenomena which have not yet been observed. They all contain various numbers of very massive vector mesons and most predict several new massive leptons and/or hadrons. The higher order corrections to the weak interactions when they are calculable are finite and very small. To determine if any of these theories correctly describes the weak interactions, the most likely place to look is in the purely leptonic sector where strong interactions do not introduce complications. However, these experiments are difficult and hard to come by.

Aside from the lack of experimental corroboration, none of the gauge models is entirely compelling in its description of weak interaction phenomenology. Most models describe some aspect of weak phenomenology, but none can account

for all the properties of the weak interactions in an entirely natural manner. It is also a matter of taste what quantities we require a model to predict. Some of the more perplexing aspects of the weak interactions we might want a successful model to explain are:

a) the role of the muon. Why is the muon much heavier than the electron and does it play a necessary role in particle dynamics? Is the ratio of the electron mass to the muon mass calculable? (To good approximation $m_e/m_\mu \approx 2\alpha/3$)[18]

b) the Cabibbo angle. This is easily accommodated in many models but not readily explained or calculable. In models where it can be calculated, it is difficult to get nontrivial ($\Theta \neq 0^\circ, 90^\circ$) results. Is the Cabibbo angle spontaneously generated?[19]

c) CP violation. Why is it so small; why is it only observed in the neutral kaon sector; is it perhaps spontaneously generated?[20]

d) the $|\Delta I|=1/2$ rule. Perhaps gauge theories can explain this selection rule which is not well understood.[21]

e) the proton-neutron mass difference. Can we calculate this and can we obtain the correct sign? (see Chapter 4).[22]

There are important questions which remain to be solved which are not phenomenological in nature. Although the Higgs mechanism successfully accomplishes the spontaneous symmetry breaking, it seems contrived. Some have considered the possiblity that the symmetry breaking occurs dynamically without the introduction of extraneous scalar fields. However it is not known how to calculate dynamical symmetry breaking or where the extra degree of freedom for the massive vector meson field comes from.[23]

Another important group of problems stems from the strong interactions. Whether or not the strong interactions are to be described by a gauge theory, there seems to be a conflict between our previous ideas of symmetries and

symmetry breaking and the notions of symmetry breaking in gauge theories. In conventional theories, the Lagrangian is nearly invariant under global symmetry groups (i.e. SU(3), SU(3)xSU(3)) which are broken by small noninvariant terms in the Lagrangian. However, the only kind of symmetry breaking which can be tolerated in gauge theories is spontaneous symmetry breaking. Actual symmetry breaking terms in the Lagrangian, however small, destroy the renormalizability of the theory. In this context, it is obscure what, if any, connection exists between approximate strong interaction invariances and gauge symmetries. If we accept the viewpoint of gauge symmetries, we must explain what the pion is and why it satisfies low energy theorems (see Chapter 4 for a possible explanation).

Although the problems to be solved are formidable, the prospects are heady. After the initial euphoria which infected the physics community with the discovery of a strategy for constructing a renormalizable weak interaction theory, physicists-theorists and experimentalists alike-are settling in to tackle these difficult problems.

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3. CONSTRAINTS ON HIGGS FIELDS IN UNIFIED WEAK-ELECTROMAGNETIC GAUGE THEORIES

I. Introduction and Summary

The development of renormalizable gauge theories which attempt to unify the weak and electromagnetic interactions[1] has opened a Pandora's box of possible models. Once a gauge group is chosen, the model builder must decide on a representation of scalar Higgs particles, whose nonzero vacuum expectation values (VEV) spontaneously break the gauge symmetry, and assign known and probably unknown fermions to group multiplets in such a way that he satisfies the constraints imposed by the experimental cross sections, masses, moments, decay rates, etc.

In this chapter, we present simple criteria for determining, given the underlying group, which representations of the Higgs particles can reproduce a vector meson mass spectrum in which one vector meson, the photon, remains massless and all other vector mesons acquire mass. Kibble has discussed the choice of Higgses in the context of the strong interactions, thus requiring that the gauge symmetry be completely broken and that all vector mesons acquire mass.[2] However, his analysis does not apply when the photon is one of the vector mesons of the theory, for two reasons. One is obviously the m=0 constraint. The other reason, which is intimately related to the first, is that the electric charge Q associated with the photon is the generator of a U(1) symmetry which remains unbroken after the Higgses acquire VEV. Therefore, we must ask more specifically: given the gauge group G and the electric charge Q, when does a Higgs representation "work," that is give the desired spectrum?

Alternatively, given the Higgs representation, we may then ask what restrictions are placed on the charge operator of the theory. Naturally, the choice of charge operator influences the assignment of leptons and hadrons to multiplets. For example, in an SU(3) model, we may ask for what Higgs representation can the charge be the usual quark charge operator (cf. section IV for an answer).

Our method is incomplete in two respects. We assume that representations are not self adjoint, which may make a difference for those representations which may be self adjoint (cf. section IV for an example). We also assume the Higgs VEV are free parameters, whereas they are constrained to minimize the potential. Because the renormalizability of the theory requires that the potential be at most a quartic gauge invariant polynomial in the Higgs fields, the VEV in general cannot be chosen at will. Therefore the self-consistency of the renormalization scheme may force the VEV to be invariant under a larger group than the symmetry group left unbroken if the VEV parameters could be taken arbitrarily.[3]

In section II we review briefly the necessary group theory apparatus. In section III we describe the solution to the problem and in section IV illustrate the method for all the representations and possible charge assignments for SU(3) and SU(3)xU(1). In the appendix, we give the results for all unexceptional classical Lie groups. We find that for any group, aside from a small number of low dimension Higgs representations, which do not give the correct mass spectrum, all other representations are acceptable from this standpoint (cf. eqtns IV.1 and the appendix). For example, in SU(n) the two n dimensional representations (inequivalent for $n>2$) do not work. Neither does the n^2-1 dimensional adjoint representation if it is taken to be self-adjoint. This is perhaps of interest because simplicity will lead one to choose the Higgs representation to be "as small as possible"—in fact, this is not always possible.

Of course, checking the vector meson masses is only the first step in constructing a theory. But we trust that our method lightens the load of model builders and saves them the task of calculating ugly determinants.

II. Remembrance of Things Past

Given a Lie group G of order g and rank ℓ, we can express the group algebra in the Cartan-Weyl canonical form.[4] There are ℓ diagonal generators H_i which commute among themselves and $\frac{g-\ell}{2}$ pairs of generators E_α, $E_{-\alpha}$ such that $E_\alpha^+ = E_{-\alpha}$ and

$$[H_i, E_\alpha] = \alpha_i E_\alpha$$

The ℓ dimensional vectors α_i are the roots of the group. The algebra is closed with the remaining commutation relations:

$$[E_\alpha, E_\beta] = N_{\alpha\beta} E_{\alpha+\beta} \text{ if } \alpha+\beta \text{ is a nonzero root}$$

$$[E_\alpha, E_{-\alpha}] = \alpha^i H_i \text{ where } \alpha^i = g^{ik}\alpha_k$$

and

$$g^{ik} = \sum_\alpha \alpha_i \alpha_k .$$

The root vectors uniquely characterize any infinitesimal Lie group. A diagram of the root vectors in their ℓ dimensional space is called the vector diagram of the group.

An irreducible representation of dimension N is given if we have g NxN irreducible matrices which satisfy the group commutation relations. Consider the N dimensional eigenvectors of the central operators H_i: $H_i u = m_i u$. The ℓ-vectors $(m_1, \ldots, m_\ell)$ are called the weights of the representation. The weights completely determine a representation and have some useful properties. If m is a weight and α any root, then $\vec{m} - (2(m\cdot\alpha)/\alpha\cdot\alpha)\vec{\alpha}$, the weight formed by reflection with respect to the hyperplane perpendicular to $\vec{\alpha}$, is also a weight of the same multiplicity. The set of reflections in hyperplanes perpendicular to roots is a group, the Weyl group. Weight points connected by Weyl reflection are said to be equivalent.

We can define an ordering of weights such that $\vec{m}$ is higher than $\vec{m}'$ if the first nonzero component of their difference is positive. The highest weight, which is always nondegenerate, completely determines the irreducible representation. In fact, for a group of rank ℓ, there are ℓ fundamental dominant weights $\vec{L}^{(1)}, \ldots, \vec{L}^{(\ell)}$ such that any irreducible

representation has highest weight $\vec{L} = \Sigma n_i \vec{L}^{(i)}$, n a non-negative integer, and for any such $\vec{L}$, there exists an irreducible representation with $\vec{L}$ as its highest weight.

III. Solution of the Problem

Consider a gauge theory associated with a Lie group G. The Higgs particles in the theory belong to a representation (not necessarily irreducible) of the group with representation matrices $\{T_i, i=1, \ldots ,g\}$ which satisfy the group multiplication law

$$[T_i,T_j] = ic_{ijk}T_k$$

where c_{ijk} are the group structure constants.

The part of the Lagrangian containing only vector mesons and Higgses is:

$$\mathcal{L} = -\tfrac{1}{4}|\partial_\mu V^a_\nu - \partial_\nu V^a_\mu - gc^{abc}V^b_\mu V^c_\nu|^2$$
$$-\tfrac{1}{2}|(\partial_\mu + igT^a \cdot V^a_\mu)\phi|^2 + V(\phi)$$

$V(\phi)$ is at most a quartic polynomial in the Higgs fields. Its minimum at $\langle\phi\rangle=\eta$

$$\left\langle\frac{\partial V}{\partial\phi}\right\rangle\Big|_{\langle\phi\rangle=\eta}=0 \quad ; \quad \left\langle\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right\rangle\Big|_{\langle\phi\rangle=\eta}>0$$

determines up to group rotation the possible VEV's of the Higgs fields. We assume that $V(\phi)$ can be rigged to give whatever $\langle\phi\rangle$'s we choose.

In the tree approximation, the vector meson mass matrix is

$$M^2_{ab} = g^2\langle T^a\phi\rangle^* \cdot \langle T^b\phi\rangle$$

In general, we have to include the adjoint Higgses which are the anti-particles and correspond to representation matrics $\bar{T}_i = -T^*_i$ and VEV $\langle\bar{\phi}\rangle = \langle\phi\rangle^*$. Thus, for T hermitian,

$$M^2_{ab} = g^2[\langle\phi\rangle^* T_a T_b \langle\phi\rangle + \langle\bar{\phi}\rangle^* \bar{T}_a \bar{T}_b \langle\bar{\phi}\rangle]$$
$$= g^2[\langle\phi\rangle^* T_a T_b \langle\phi\rangle + \langle\phi\rangle^* T_b T_a \langle\phi\rangle]$$

Alternatively, we can define $\tilde{<\phi>}$ as a 2N dimensional <u>real</u> vector with corresponding reducible 2Nx2N representation matrices which are purely imaginary. M^2_{ab} is a semi-positive definite and symmetric gxg matrix, where g is the order of the group. By suitable choice of basis we may call the charge operator $Q=T_1$. Q is any diagonal matrix constructable from the ℓ independent diagonal central matrices of the group. We impose the physical constraint $Q<\phi>=0$, namely only neutral members of a representation can acquire VEV since the electromagnetic symmetry is unbroken. The vector meson V_1 is then the photon of the theory; its mass remains zero to all orders since the symmetry remains unbroken. We are left with the (g-1) dimensional reduced mass matrix $\hat{M}^2$ formed by crossing out the first row and column. Its eigenvalues are the mass squares of the remaining vector mesons. Therefore, the necessary and sufficient spectral condition is $\det\hat{M}^2 \neq 0$.

In the $\tilde{T}$ language, the $\tilde{T}_i\tilde{<\phi>}$ are g-1 purely imaginary 2N dimensional vectors. We can choose a set of g-1 orthonormal vectors $e_k (e_i \cdot e_k = \delta_{ik})$ such that

$$<\tilde{T}_i\tilde{\phi}> = i\sum_{j=1}^{g-1} a_{ik} e_k$$

If the $\tilde{T}_i\tilde{<\phi>}$ are linearly independent, $\det a \neq 0$; otherwise $\det a=0$. Then

$$\hat{M}^2_{ij} = -g^2 <\tilde{T}_i\phi><\tilde{T}_j\phi> = g^2(aa^T)_{ij}$$

Therefore $\det\hat{M}^2 \neq 0$ is equivalent to $\det a \neq 0$ and the g-1 vectors $\tilde{T}_i\tilde{<\phi>}$ are linearly independent. More generally, given any basis with representation matrices $X_i (X=G\tilde{T}G^{-1}$, G non-singular) $\det\hat{M}^2 \neq 0$ if and only if $V_j = X_j<\phi>$ are g-1 linearly independent vectors.

Beginning for simplicity, with a Higgs multiplet belonging to an irreducible representation with highest weight $\vec{m}$, we give a geometric construction for determining the number of massive vector mesons (or linearly independent V_j) in the theory for any Q. The construction is only easily done for low rank groups, but the same method can be applied algebraically for all groups.

In the ℓ dimensional weight space, operate on the highest weight $\vec{m}$ by all the transformations of the Weyl group to form the envelope of the representation, the polyhedron formed with the highest weight and its equivalents as vertices. All other points in the representation are obtained by displacing the highest weight and its equivalents by any sum of root vectors which remains within or on the envelope. The multiplicity of a weight point (number of representation elements with the same weight) is given iteratively by Cartan.[5] After constructing the weight diagram of a representation, its adjoint is simply obtained by reflection through the origin in the weight space.

Any ℓ-1 dimensional hyperplane through the origin defines a plane Q=0. Mark with an x all points on the plane Q=0. For each root vector, put a box around any point which any xed point connects to with that root. For each of the (ℓ-1) central operators H_i (eliminating Q) put a box around any xed point which is not in the plane H_i=0. If this procedure can be followed such that no point is boxed twice, then since each point in the weight diagram is linearly independent there will be g-1 massive vector mesons and one massless one. If there is no boxing procedure which gives g-1 independent boxed points, the maximum number of such points is the number of massive vector mesons.(see Figure 2 for some examples). There may be accidental degeneracies such that in fact the number of massive particles is smaller for a particular choice of VEV's. However, some choice of VEV will give the maximum number.

For any group, we can define the notion of classes of representations. Two representations are in the same class if the weight points of one representation are connectible to the weight points of the other by sums of root vectors. There will be a finite number of such classes. If two representations are in the same class, the points of the representation with smaller highest weight are interior points of the higher representation. Therefore if for some Q, the mass spectrum is correct for the lower representation, then it automatically works for all higher representations in the same class. This greatly reduces the amount of work involved in finding which Higgs representations will do the job for any group.

Note that the procedure can be used to determine which vector mesons are heavy and which superheavy for various assignments of VEV. For example, if one point has VEV much larger than any of the others, then those vector mesons corresponding to operators which connect this point to another point in the representation will be superheavy. This method is easily extended to several Higgs multiplets if we include all representation and adjoint points in the weight space and proceed as before.

IV. Examples: SU(3) and SU(3) x U(1)

As examples, we show how our method can be applied to all representations of SU(3) and SU(3) x U(1).

The root vectors of the group SU(3) are the vectors $e_i - e_j$, i, j = 1,2,3, $i \neq j$. All roots lie in the plane $\sum_{i=1}^{3} x_i = 0$ and the group is of dimension 8 and rank 2. Any representation can be defined by the Young tableau (f_1, f_2); it is a tensor of rank $f_1 + f_2$ whose highest weight is $1/3(2f_1 - f_2, -f_1 + 2f_2, -f_1 - f_2)$. The Weyl group is the group of permutations of the weight components.

Since the weight points all lie in a plane, it is convenient to parameterize the weight plane in terms of the vectors $e_1 - e_2$ and $e_2 - e_3$ which are at 120° to one another. The components $m_1 - m_2$ and $m_2 - m_3$ are integers. The SU(3) representations fall into three classes, {(0,0); (2,1); (3,0) ...}, {(1,0); (2,2); (3,1)}, {(1,1); (2,0); (3,2); ...} where $f_1 + f_2 \equiv 0, 1, 2 \pmod 3$. Our results are as follows:

(i) singlet (0,0) and triplets (1,0), (1,1) do not work

(ii) sextets (2,0) and (2,2) work (1)

(iii) octet (2,1) works if it is not self-adjoint; does not work if it is self-adjoint

(iv) all higher representations work

(see Figures 1 and 2)

We can also determine which charges work for a given representation. Any Q such that Q=0 includes a weight point, whose components are not all

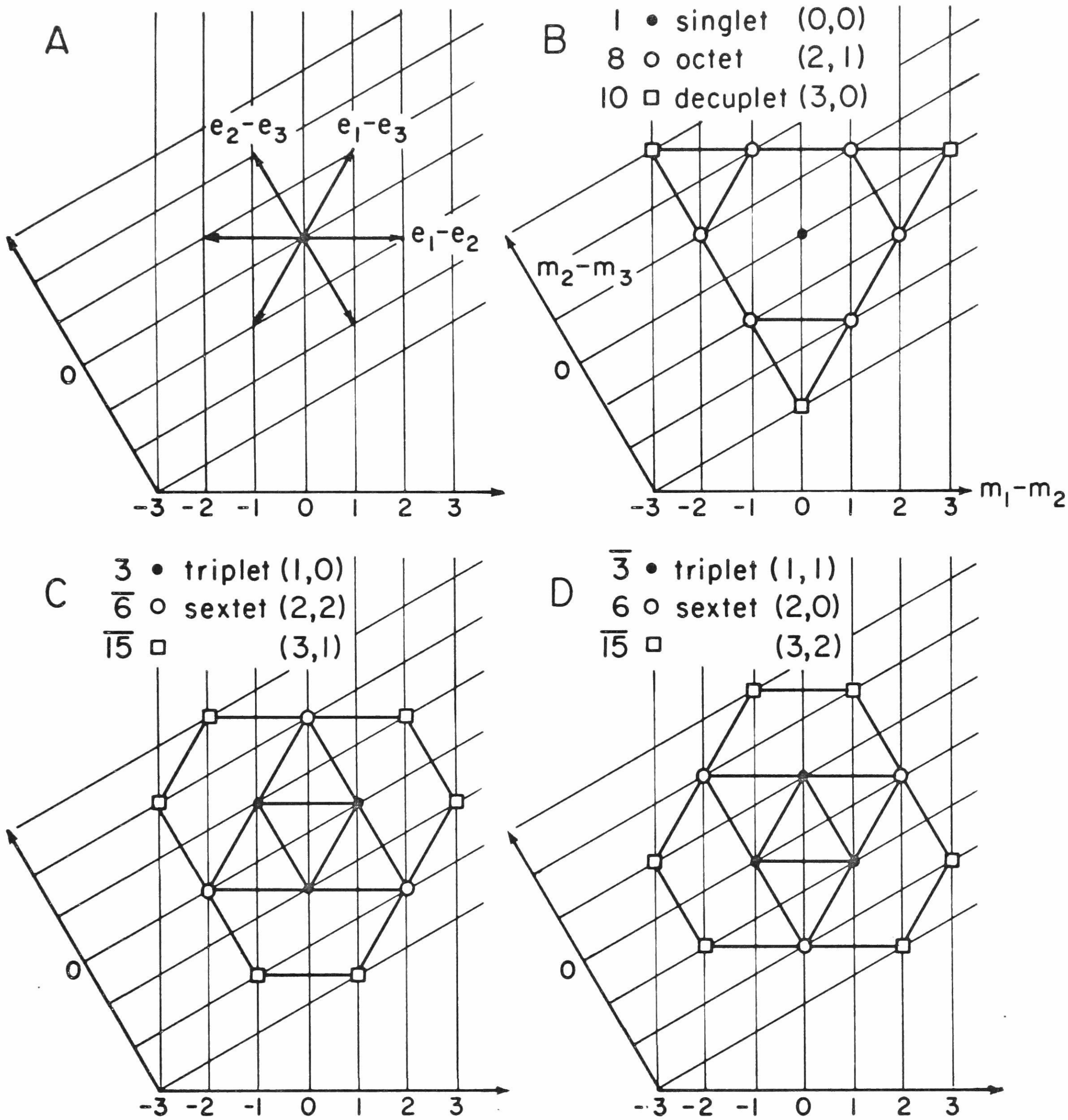

Figure 1. Root space and Weight space for SU(3)

A. Root vectors for SU(3)

B. Class I Representations : $f_1 + f_2 \equiv 0 \pmod 3$

C. Class II Representations : $f_1 + f_2 \equiv 1 \pmod 3$

D. Class III Representations : $f_1 + f_2 \equiv 2 \pmod 3$

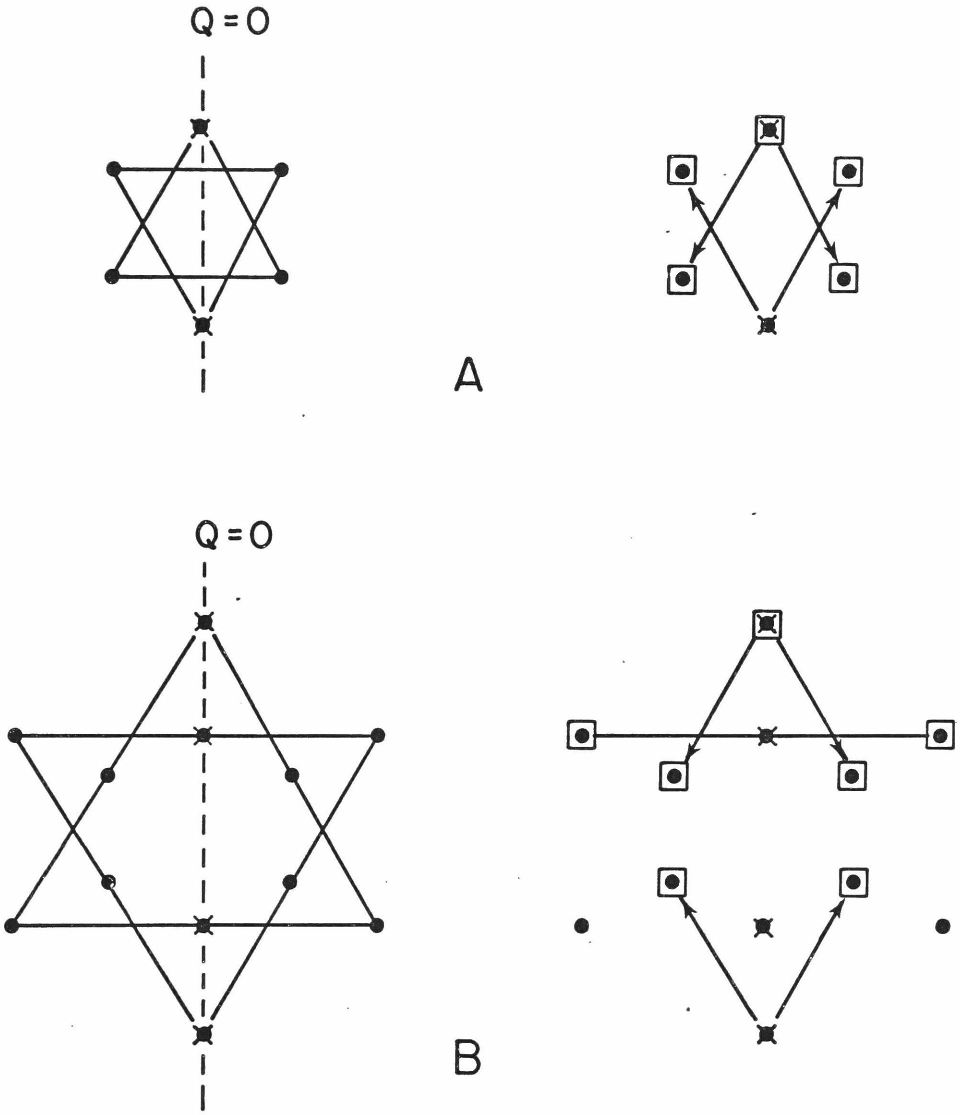

Figure 2. Illustration of the Method

A. SU(3) triplet does not work (only 5 massive vector mesons)

B. SU(3) sextet works (7 massive vector mesons)

zero, in the interior (not on the envelope) clearly works [i.e. the weight point (1,0) in the 15 rep (3,1)]. So does any Q=0 which intersects a point on the envelope which is not the apex of a triangular representation (if $f_2 = 0$ or $f_1 = f_2$ the envelope is triangular instead of hexagonal). In a triangular representation, the envelope makes a 60° angle at the apex; since the root vectors are at angles of 60° to one another, only two of the three root vector directions connect to weight points in the representation. However, for $f_1 \geqslant 2$ the line through a triangular apex always contains at least one other non-apex point in the representation.

All that remains is to characterize the points in the representation weight space. They are all points of the form

I. $(-f_1+f_2-k+2j_k,\ f_1-k-j_k)$ where k is an integer in the interval $(0,\ f_2)$ and j_k an integer in the interval $(0,\ f_1-f_2+k)$

II. $(-f_1+\ell+2j_\ell, f_1-f_2-2\ell-j_\ell)$ where ℓ is an integer in $(0,\ f_1-f_2)$ and j_ℓ is an integer in $(0,\ f_1-\ell)$.

with corresponding allowed Q's (exclude $Q \equiv 0$),

$$Q_I = (f_1-k-j_k)\ (H_1-H_2) + (f_1-f_2+k-2j_k)(H_2-H_3)$$

$$Q_{II} = (f_1-f_2-2\ell-j_\ell)(H_1-H_2) + (f_1-\ell-2j_\ell)(H_2-H_3)$$

In particular, it may be interesting to ask for what $(f_1,\ f_2)$ can Q be proportional to the quark charge matrix $\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$? In that case we are restricted to either I. $\frac{f_1+f_2}{3}$ an integer and $f_1 \leqslant 2f_2$ or II. $\frac{f_1-2f_2}{3}$ an integer and $f_1 \geqslant 2f_2$. The simplest examples are the <u>nonself-adjoint</u> octet (2,1) and the decuplet (3,0).

The case of SU(3) x U(1) is more interesting physically. Since the weak interactions couple only to the left handed part of leptons while charge couples to both right and left handed pieces, it is hard

to build models in which both the charge and the weak generators are in the same SU(3) group. The SU(3) x U(1) weight diagrams are now 3-dimensional with the addition of a perpendicular dimension corresponding to the U(1) direction associated with operator H_0. The root vectors are as before with zero component in the U(1) direction (since it is an abelian subgroup). All Higgses work except the singlet and a triplet (but two triplets with different H_0 quantum number will do it).

The charge analysis reduces to two cases. If the U(1) quantum number is zero, then any $[(f_1, f_2);0]$ except singlet or triplet works with $Q=H_0$ the only possible choice of Q. If the U(1) quantum number is not zero we can take it to be 1 by scaling H_0. By considerations similar to those for SU(3), any charge operator Q for which the representation contains neutral points will give the correct mass spectrum, except those Q's whose only neutral elements lie on the H_0 axis and those which contain only an apex point in a triangular representation.

Write $Q = aH_0 + b(H_1-H_2) + c(H_2-H_3)$. If $a = 0$, then the same Q's which work for SU(3) work here. If $a \neq 0$, take $a = 1$ by scaling Q. Then $Q = H_0 + b(H_1-H_2) + c(H_2-H_3)$ gives the correct spectrum for any b,c such that either

$$1 + b(-f_1+f_2-k+2j_k) + c(f_1-k-j_k) = 0$$

$$\text{or} \quad 1 + b(-f_1+\ell+2j_\ell) + c(f_1-f_2-2\ell-j_\ell) = 0$$

with the following exceptions:

a) If the representation is $[(f_1,0);1]$ then $Q = H_0-\frac{1}{f_1}(H_1-H_2) + c(H_2-H_3)$ [and the two other Q's obtain by cyclic permutations of (1,2,3)] gives the correct spectrum only if there exists more than one (k, j_k) or (ℓ, j_ℓ) such that

$$1 - \frac{1}{f_1}(-f_1-k+2j_k)+c(f_1-k-j_k) = 0$$

$$\text{or} \quad 1 - \frac{1}{f_1}(-f_1+\ell+2j_\ell)+c(f_1-2\ell-j_\ell) = 0$$

Thus in the sextet (2,0), $Q = H_0 - 1/2(H_1-H_2)+c(H_2-H_3)$ gives the correct spectrum only for the special cases $c = -1$ and $c = 1/2$.

b) If the representation is $[(f_1,f_1); 1]$, then $Q_0 = H_0 + \frac{1}{f_1}(H_1 - H_2) + c(H_2 - H_3)$ (and Q's obtained by cyclic permutation) give just one massless vector meson only if there exists more than one (k, j_k) or (ℓ, j_ℓ) such that

$$1 + \frac{1}{f_1}(-k+2j_k) + c(f_1 - k - j_k) = 0$$

or

$$1 + \frac{1}{f_1}(-f_1 + \ell + 2j_\ell) + c(-2\ell - j_\ell) = 0$$

This analysis can be applied, for example, to show that in the Schechter and Ueda SU(3) x U(1) model with $Q = H_0 + 1/3(^2 - 1_{-1})$, they can choose any Higgs multiplet except the singlet or triplet.[6]

Appendix

In this Appendix, we solve the Higgs problem for the classical Lie groups to illustrate the general method when geometric construction is no longer practical. The classical Lie groups belong to the four families A_ℓ $[SU(\ell+1)]$, $B_\ell[O(2\ell+1)]$, $C_\ell[Sp(2\ell)]$, and $D_\ell[O(2\ell)]$. ℓ is the rank of the group.

I. A_ℓ $(\ell \geqslant 1)$: The root vectors, $\{(e_i-e_j)\ i,j = 1, \ldots, \ell+1\}$ all lie in the ℓ dimensional hyperplane $\sum_{i=1}^{\ell+1} x_i = 0$. The order of the group is $(\ell+1)^2 - 1$. In the weight space, the Weyl group consists of all permutations of the weight components. The fundamental dominant weights are

$$\vec{L}^{(1)} = \left(\frac{\ell}{\ell+1}, \frac{-1}{\ell+1}, \ldots, \frac{-1}{\ell+1}\right)$$

$$\vec{L}^{(2)} = \left(\frac{\ell-1}{\ell+1}, \frac{\ell-1}{\ell+1}, \frac{-2}{\ell+1}, \ldots, \frac{-2}{\ell+1}\right)$$

$$\cdots$$

$$\vec{L}^{(\ell)} = \left(\frac{1}{\ell+1}, \frac{1}{\ell+1}, \ldots, \frac{1}{\ell+1}, \frac{-\ell}{\ell+1}\right)$$

Let the highest weight of a representation be $\vec{L} = \sum_{i=1}^{\ell} k_i \vec{L}^{(i)}$. There are $\ell+1$ classes of representations, $C_{(\ell+1-r)}$, $r=0, \ldots, \ell$. A representation is in $C_{(\ell+1-r)}$ if its weight $\vec{L} = \frac{1}{\ell+1}(\ell_1, \ell_2, \ldots, \ell_{\ell+1})$ satisfies $\ell_i \equiv r (\mathrm{mod}\ \ell+1)$. Clearly the fundamental representations $L^{(i)}$ belong to the class $C_{(i)}$ and more generally $\vec{L}$ belongs to $C_{(j)}$ where $\sum_{i=1}^{\ell} k_i\, i \equiv j (\mathrm{mod}\ \ell+1)$.

Denote by $\vec{L}^{(i)}_{min}$ the smallest weight in $C_{(i)}$ which gives the correct mass spectrum. For $C_{(\ell+1)}$, $\vec{L}^{(\ell+1)}_{min} = (100 \ldots -1)$. For C_ℓ, $\vec{L}^{(\ell)}$ does not work but $\vec{L}^{(\ell)}_{min} = \vec{L}^{(\ell-1)} + \vec{L}^{(1)} = \left(\frac{\ell+2}{\ell+1}, \frac{1}{\ell+1}, \ldots \frac{1}{\ell+1}, \frac{-\ell}{\ell+1}, \frac{-\ell}{\ell+1}\right)$ does. In general, $\vec{L}^{(j)}_{min} = \vec{L}^{(\ell)} + \vec{L}^{(j+1)}$ for $j = 1, 2, \ell-1$. For $3<j<\ell-2, \vec{L}^{(j)}_{min} = \vec{L}^{(j)}$.

For example, for SU(4) all representations except the singlet and the three fundamental representations (two of dimension 4, one of dimension 6) will do the trick.

II. $B_\ell (\ell \geq 1)$: The Group B_ℓ of order $\ell(2\ell+1)$ has roots $R_\ell = \{\pm e_i, \pm e_i \pm e_k, i,k = 1, \ldots, \ell, i \neq k\}$. The Weyl group consists of all permutations together with any sign change. The fundamental weights are:

$$\vec{L}^{(1)} = (1/2\ 1/2\ \ldots\ 1/2)$$

$$\vec{L}^{(2)} = (100\ \ldots\ 0)$$

$$\ldots$$

$$\vec{L}^{(\ell)} = (11\ \ldots\ 10)$$

There are two classes of representations--integral and half integral. For the integral representations $\vec{L}_{min} = (110\ \ldots\ 0)$. For the non-integral representations $\vec{L}_{min} = (3/2\ 1/2\ \ldots\ 1/2)$ for $\ell < 6$ and $\vec{L}_{min} = (1/2\ 1/2\ \ldots\ 1/2)$ for $\ell \geq 6$. For example, for O(5) only the two fundamental representations (of dimension 4 and 5) and the singlet do not work.

III. $C_\ell (\ell \geq 2)$: The group C_ℓ is of order $\ell(2\ell + 1)$ and its roots are $R_\ell = \{\pm 2e_i, \pm e_i \pm e_k\ i,k = 1, \ldots, \ell\}$. The Weyl group is the same as for B_ℓ, but the weight components can only be integers. The fundamental weights are

$$\vec{L}^{(1)} = (100\ \ldots\ 0)$$

$$\vec{L}^{(2)} = (110\ \ldots\ 0)$$

$$\ldots$$

$$\vec{L}^{(\ell)} = (11\ \ldots\ 1)$$

There are two classes of representations--C_{even}, those representations for whose weights $\vec{L} = (\ell_1, \ldots, \ell_\ell)$, $\sum_{i=1}^{\ell} \ell_i$ is even, and C_{odd}, those whose sum is odd. For C_{even}, if $\ell \geq 4$, $\vec{L}^{even}_{min} = (11110\ \ldots\ 0)$ and if $\ell < 4$, $\vec{L}^{even}_{min} = (20\ \ldots\ 0)$. For C_{odd}, if $\ell < 4$, $\vec{L}^{odd}_{min} = (2,1,0\ \ldots\ 0)$ and if $\ell \geq 4$, $\vec{L}^{odd}_{min} = (1110\ \ldots\ 0)$. Therefore, for Sp(4) only the singlet and

two fundamental representations which are of dimension 4 and 5 do not work. This checks with the result in B_ℓ for $O(5)$ since $Sp(4) \cong O(5)$. For $Sp(6)$, the singlet and three fundamental representations of dimension 6, 8, and 14 are not acceptable.

IV. $D_\ell(\ell > 2)$: The group D_ℓ is of order $\ell(2\ell - 1)$ and its root vectors are $R_\ell = \{\pm e_i \pm e_k, \; i, k = 1, \ldots, \ell, \; i \neq k\}$. The Weyl group consists of all permutations together with all changes of sign in pairs. The weight components are either all integers or all half integers. The fundamental weights are:

$$\vec{L}^{(1)} = (1/2 \; 1/2 \; \ldots \; 1/2)$$

$$\vec{L}^{(2)} = (1/2 \; 1/2 \; \ldots \; 1/2 \; - 1/2)$$

$$\vec{L}^{(3)} = (10 \; \ldots \; 0)$$

$$\cdots$$

$$\vec{L}^{(\ell)} = (11 \; \ldots \; 1100)$$

There are four classes of representations. If $\vec{L} = (\ell_1, \ldots, \ell_\ell)$ is the highest weight, the representation is in $C_{(1/2,\,\mathrm{even})}$ if ℓ_i are half integral and $(2\Sigma\ell_i - \ell)/2$ is even; in $C_{(1/2,\,\mathrm{odd})}$ if ℓ_i are half integral and $(2\Sigma\ell_i - \ell)/2$ is odd; in $C_{(0,\,\mathrm{even})}$ if ℓ_i are integers and $\Sigma\ell_i$ is even; and in $C_{(0,\,\mathrm{odd})}$ if ℓ_i are integers and $\Sigma\ell_i$ is odd. For $C_{(1/2,\,\mathrm{even})}$, if $\ell \geqslant 8$ $\vec{L}_{min} = (1/2 \; 1/2 \; \ldots \; 1/2)$, otherwise $\vec{L}_{min} = (3/2 \; 1/2 \; \ldots \; 1/2 \; - 1/2)$; for $C_{(1/2,\,\mathrm{odd})}$, if $\ell \geqslant 8$ $\vec{L}_{min} = (1/2 \; 1/2 \; \ldots \; - 1/2)$, otherwise $\vec{L}_{min} = (3/2 \; 1/2 \; \ldots \; 1/2)$. For $C_{(0,\,\mathrm{even})}$ if $\ell \geqslant 4$, $\vec{L}_{min} = (11110 \; \ldots \; 0)$ and for $\ell = 3$, $\vec{L}_{min} = (110)$; for $C_{(0,\,\mathrm{odd})}$, $\vec{L}_{min} = (1110 \; \ldots \; 0)$ for all ℓ. Thus for $O(6)$, all representations except the singlet and the 4,4, and 6 dimensional fundamental representations work. This checks with the result for $SU(4)$ which is isomorphic to $O(6)$.

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$V(\phi)$ overlooks a possible solution. They look at a self adjoint Higgs octet in an SU(3) model. The most general SU(3)-invariant potential is then $V(\phi) = \frac{1}{4}\alpha_1 Tr(\phi^4) + \frac{1}{3}\alpha_2 Tr(\phi^3) + \frac{1}{2}\alpha_3 Tr(\phi^2)$. If $\alpha_2 \neq 0$, as they correctly point out, the most general $<\phi>$ which minimizes $V(\phi)$ leaves an SU(2) x U(1) subgroup invariant and therefore gives 4 massless vector mesons. If we however look at $\alpha_2 = 0$ (which can be naturally implemented by imposing the reflection symmetry $\phi \to -\phi$), then only U(1) x U(1) is left invariant, which is the same group found without imposing the renormalizability constraint. If the nonself-adjoint octet is taken, only one massless vector meson remains as in the unconstrained case. Presumably if the symmetry is broken in lowest order, the breaking would persist in higher orders, although this should be checked since for renormalization we must add a counterterm proportional to $Tr(\phi^3)$ unless the additional reflection symmetry is imposed.

4. All the group theory in the chapter comes from the lucid unpublished lecture notes of G. Racah, "Group Theory and Spectroscopy," Institute for Advanced Study (1951). An abridged version is published in F. Gursey, ed., Group Theoretical Concepts and Methods in Elementary Particle Physics.
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4. THE PROTON-NEUTRON MASS DIFFERENCE AND THE PION MASS IN A GAUGE MODEL

I. Introduction

One of the most exciting features of gauge theories of the weak and electromagnetic interactions is the possibility of calculating masses and mass differences. Previously in renormalizable field theories, if a bare mass or mass difference vanished, then either it remained zero to all orders because of an underlying symmetry of the Lagrangian or it was infinite in higher orders. The infinity could be taken care of by renormalization, but in the process the ability to calculate the mass difference was lost. In a spontaneously broken gauge theory, if a mass difference or mass is zero in zeroth order for all possible coupling constants even after the symmetry is broken, then that quantity must be finite and calculable. Why is this? Since the renormalization procedure respects the symmetries of the theory, there are no possible counterterms to cancel infinites which might arise in higher orders. Therefore, since the theory is renormalizable, the calculation of higher order effects must yield a finite result. (Of course, if a zeroth order relation does not arise naturally from the theory but is put in by hand by artificially constraining parameters in the Lagrangian then there is no reason to expect finite corrections to the relation. This is because the renormalization procedure does not know about artificial relations but only about relations which follow from the gauge symmetry of the theory and the representation content of the fields.)[1]

This chapter studies particular questions in the domain of this new calculability in the framework of an SU(2) x SU(2) x U(1) model of the weak and electromagnetic interactions. The model is basically due to Weinberg but our interpretation of it is quite different.[2] One of the aims of the chapter is to study the proton-neutron mass difference which is calculable in this model. The second aim is to investigate mechanisms for incorporating pions into gauge theories.

If a gauge theory is to describe the weak and electromagnetic interactions, the gauge symmetry of the Lagrangian must be spontaneously

broken to give masses to the gauge vector bosons and some of the fermions. One mechanism for breaking the symmetry is to introduce scalar mesons, called Higgs particles, which acquire nonzero vacuum expectation value.[3] Varying the Higgs content for a given gauge group changes the way in which the symmetry is broken. Another possibility is that symmetry breaking occurs dynamically without the introduction of additional spinless fields. The idea of dynamical symmetry breaking is currently much discussed, particularly in the context of strong interaction gauge theories, where the introduction of Higgs scalars seems to cause serious problems with respect to asymptotic freedom. In dynamically broken theories, it is anticipated that scalar particles such as pions emerge as bound states of fermions. But as yet, there has been no computational implementation of these ideas.[4]

In this chapter we exploit the Higgs mechanism to break the gauge symmetry. In varying the Higgs content in our SU(2) x SU(2) x U(1) model, we explore the effects on calculable quantities and on the physical interpretation of the model of changing the way in which the symmetry is broken. We also investigate the appearance of pions in the weak and electromagnetic sector as part of the Higgs system. In spite of their association with the weak sector, the pions interact strongly with the nucleons via a Yukawa coupling.

The sign of the proton-neutron mass difference has troubled particle physicists for decades. Previous calculations based on electromagnetism resulted in either an infinite mass difference or else the wrong sign. However, the discovery of gauge theories provided the possibility that both the weak and electromagnetic interactions could together produce a neutron which is more massive than the proton. The Weinberg SU(2) x SU(2) x U(1) model was created as an example of a gauge model in which $\Delta m|_{p-n}$ could be calculated. However, it was commonly held that in this model, as in other models based on the SU(2) group, the one loop calculation of Δm necessarily gave the wrong sign.[5] However, our study shows that the sign of the proton-neutron mass difference is a function of the Higgs content. We exhibit a possible symmetry breaking which produces a neutron which weighs more than the

proton of the theory. The point of the calculation is not so much that one should take one Higgs more seriously than another, but rather to emphasize that a lot of physics lies in the Higgs sector of a theory. (This dependence of calculable quantities on the symmetry breaking may well carry over into theories in which the symmetry is broken dynamically.)

The second part of this chapter is concerned with understanding pions in the context of gauge theories. Before the interest in gauge theories, it was often believed that the smallness of the pion mass is due to the spontaneous breakdown of a global chiral SU(2) x SU(2) symmetry. Here the pion is considered as a Goldstone boson; it has nonzero mass because the SU(2) x SU(2) symmetry is only approximately true. However, in spontaneously broken gauge theories, the Goldstone theorem is evaded via the Higgs mechanism--the would be zero mass Goldstone scalars get "eaten up" to become the longitudinal modes of the massive Yang-Mills fields. From this viewpoint it is difficult to understand what the pion is and why its mass is so small.

There have been several attempts to integrate pions into weak and electromagnetic gauge theories.

1) Hagiwara and Lee put pions into the Higgs sector in the Weinberg SU(2) x U(1) model.[6] However, as was stated by its authors, their model is artificial in the technical sense defined in their paper: parity and isotopic spin symmetry in the πN coupling are approximate and depend on setting two coupling constants almost equal to one another. This approximate equality is not stable under renormalization since it does not follow from any symmetry argument. In general even if parity is a natural strong interaction symmetry in a model, it may be difficult to guarantee that parity is not broken to order α when there are strongly interacting scalar fields in the Lagrangian.[7]

2) Weinberg's theory of pions as pseudoGoldstone bosons is a mechanism for producing spinless mesons which are massless in zeroth order but pick up finite calculable masses in higher order.[8] If the potential V of the model is forced because of gauge invariance and

renormalizability to be invariant under a larger group than the gauge group, then the model contains pseudoGoldstone bosons. These masses appear to be of order $m^2 \sim \alpha\mu^2$ (μ a typical vector meson mass), which may be too large unless they are numerically damped.

Suppose the gauge group is G, but the potential is invariant under the larger group $\bar{G}$. When the Higgs fields acquire vacuum expectation value, the potential remains invariant under a subgroup $\bar{S}$ of $\bar{G}$ for all values of parameters in the Lagrangian[9] and the Lagrangian remains invariant under U(1), the electromagnetic gauge invariance. For each generator of $\bar{G}$ not in $\bar{S}$, there is a scalar meson whose mass is zero in zeroth order. Those mesons corresponding to generators of the true symmetry group G are the true Goldstone bosons of the theory. They become the longitudinal components of the massive vector mesons. Those mesons corresponding to generators of $\bar{G}$ neither in $\bar{S}$ nor in G are the pseudoGoldstone bosons (see Figure 1). In Weinberg's theory the pseudoGoldstone bosons can be either fundamental fields in the Lagrangian or bound states. Attractive though Weinberg's idea is, there have been no models implementing it in the context of unified weak and electromagnetic interactions. (Bars and Lane, however, have a model utilizing the pseudoGoldstone mechanism in a strong interaction gauge theory.[10])

In our variation of the Weinberg SU(2) x SU(2) x U(1) model, we have a mass degenerate pseudoscalar triplet, with charges +, o, -, which interacts strongly with nucleons as pions do in the SU(2) x SU(2) σ model. Isotopic spin and parity is a natural strong interaction symmetry in this model. The triplet, which arises out of linear combinations of the Higgs fields, we identify with the pion triplet. Since it is degenerate in zeroth order, the mass difference $\delta m^2 = m^2_{\pi+} - m^2_{\pi^0}$ is calculable. We find that δm^2 is of order $\alpha\mu^2$ where μ is a typical vector meson mass. Although it is possible that numerical factors could damp δm^2 by order of 10^{-2}, the estimate is still too large for the pion mass difference.

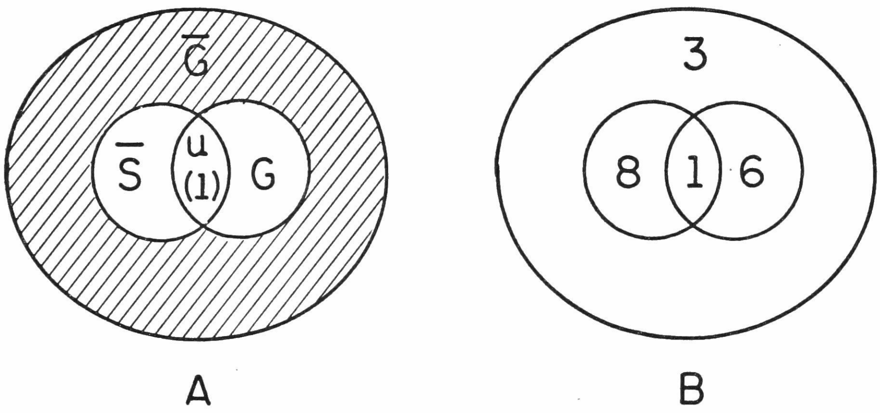

Figure 1. A) PseudoGoldstone bosons correspond to number of generators in $\bar{G}$ $-(\bar{S} \cup G)$

B) Pions as pseudoGoldstone bosons in our model. $G=SU(2)\times SU(2) \times U(1)$. $\bar{G} = O(4) \times O(4) \times O(4)$. $\bar{S} = O(3) \times O(3) \times O(3)$.

We shall show that if we modify our model by imposing a reflection symmetry on the Lagrangian, we enlarge the symmetry group $\bar{G}$ of the potential. We thus have an opportunity to explore the pseudoGoldstone mechanism in some detail. In this version of the model, $\bar{G}$ is $O(4) \times O(4) \times O(4)$. When the Higgses acquire vacuum expectation value, the potential symmetry is broken down to $O(3) \times O(3) \times O(3)$. Nine scalar fields have zero mass in zeroth order: of these six are true Goldstone bosons and three are pseudoGoldstone bosons. The pseudoGoldstone bosons are the same linear combinations of Higgs fields which we call pions in the non-pseudoGoldstone realization of the model. However, now they are massless in zeroth order and therefore we can calculate not only their mass difference, but also their masses. As before, they interact with nucleons as in the σ model with strong Yukawa coupling g_π.

When we perform the mass calculation in the one loop approximation, only the charged members of the triplet acquire mass while the neutral pion remains massless. Thus in our model the mass differences of pseudoGoldstone bosons are of the same order as the pseudoGoldstone boson masses. We believe that this phenomenon may be more general than for the specific realization considered here and may be a property of pseudoGoldstone mechanisms embedded in unified weak-electromagnetic gauge theories. This is not a bad approximation for the π-K mass difference, but is not acceptable for the pion triplet.

Compared with the mass difference problem, the order of magnitude of the pseudoGoldstone masses does not seem an insurmountable obstacle. The value of m^2 can be damped by numerical factors (in our model damping by factors of 10^{-2} is not implausible; in that case $m^2 \sim 10^{-2}\alpha\mu^2$ which is reasonable). However, if we consider our calculation of $m^2_{\pi+}$ as an estimate for δm^2 (since $m^2_{\pi 0}$ is zero) then we again have a result which is too large for the pion triplet. Another possible way around the large estimate for the pseudoGoldstone masses is uncovered by our calculation. The one loop calculation of m^2 may be zero for some of the pseudo-Goldstone masses as it is for our neutral pion. In that case m^2 for those mesons would be nonzero only in the two loop approximation and

hence presumably of order $m^2 \sim \alpha^2\mu^2$. (We could perhaps imagine a model including strange particles in which in the one loop approximation the pion triplet remained massless while the K and η picked up mass.)

We also raise the question: is there parity violation of order α? Weinberg has shown that in certain classes of models involving strongly interacting vector mesons parity violation does not occur to order α.[11] However, our model which contains strongly interacting scalar fields in the Lagrangian is not covered by his result. A preliminary investigation of the pion-nucleon form factor reveals that the parity violating piece of the one loop radiative corrections is a calculable weak effect of order $\alpha m^2_{nucleon}/\mu^2$. Our calculation only treats the strong interactions perturbatively, but we expect that the result is more generally valid.

The aim of this chapter is to study the effects of symmetry breaking on calculable masses and mass differences and on how pions can fit into gauge theories. Of course, we do not pretend that our model is realistic, but we find it an interesting model to test some of these ideas. The limitations of the model are readily apparent. It does not include strange particles and the extension to strange particles is no easy task. Perhaps a more complete model involving strangeness would give us a more realistic mass spectrum for the pseudoscalar octet realized as pseudoGoldstone bosons.

Furthermore, the complications of the strong interactions have been completely neglected in all of the calculations. Clearly the strong interactions must be treated nonperturbatively if the calculations are to be "realistic." Future work using the tools of current algebra can perhaps remedy this shortcoming.

In section II we describe the model in detail. The proton-neutron mass difference calculation is performed in section III and its dependence on the Higgs content displayed. In sectionIV we explore the two options for incorporating pions in the model: the nonpseudoGoldstone alternative in which the pion triplet has mass in zeroth order and the pseudoGoldstone realization of zeroth order zero mass pions. In section V, we look at

the question of parity violation in the pion nucleon form factor.

After the completion of this work, we discovered that S.Y. Lee, J.M. Rawls and L.-P. Yu had the same idea of using this model to examine the pseudoGoldstone mechanism.[12] We come to slightly different conclusions, however. The mentioned authors take $g_L = g_R$ (and $v'=v''$) and because of this assumption, they arrive at a lower bound on $m_{\pi+}^2$ ($m_{\pi+}^2 > \frac{3\alpha}{4\pi}\mu^2$) which causes them to rule out the pseudoGoldstone mechanism unless the pseudoGoldstone bosons acquire mass only in the two loop approximation. However, the assumption $g_L=g_R$, $v'=v''$ is natural (in the technical sense) only if the group is $O(4) \times U(1)$, in which case there is a reflection symmetry between multiplets: for every multiplet ($T_L=m$, $T_R=n$, Y) there must be a corresponding ($T_L=n$, $T_R=m$, Y). In the model at hand the reflection symmetry cannot be realized because of the (asymmetric) lepton content of the theory. Therefore, their lower bound on $m_{\pi+}^2$ cannot be consistently derived in the context of the model. On the other hand, no assumptions on g_L vs. g_R or v' vs. v'' are needed in our work.

II. The Model

We construct the model by writing down the most general renormalizable Lagrangian which is invariant under the gauge group.[13] All particles are assigned to representations of $SU(2)_L \times SU(2)_R \times U(1)$. The electric charge operator is $Q=T_{L3} + T_{R3} + Y$. The left handed leptons (electron or muon--we treat only the electron sector but the muon and its neutrino may be added to the model in the same way as the electron) form a doublet:

$$L = \binom{\nu}{e^-}_L = \frac{(1+\gamma_5)}{2}\begin{pmatrix}\nu\\ e^-\end{pmatrix} \qquad T_L = 1/2,\ T_R = 0,\ Y = -1/2$$

The right handed electron is a singlet:

$$R = e_R^- = \frac{(1-\gamma_5)}{2}\, e^- \qquad T_L = 0,\ T_R = 0,\ Y = -1$$

There are seven gauge vector mesons transforming as the adjoint representation of the group:

$\vec{A}_L^\mu$ $\qquad T_L = 1,\ T_R = 0,\ Y = 0$

$\vec{A}_R^\mu$ $\qquad T_L = 0,\ T_R = 1,\ Y = 0$

B^μ $\qquad T_L = 0,\ T_R = 0,\ Y = 1$

To this model we add the left and right handed nucleons in a symmetric manner:

$$N_L = \begin{pmatrix} p \\ n \end{pmatrix}_L = \frac{(1+\gamma_5)}{2}\begin{pmatrix} p \\ n \end{pmatrix} \qquad T_L = 1/2,\ T_R = 0,\ Y = 1/2$$

$$N_R = \begin{pmatrix} p \\ n \end{pmatrix}_R = \frac{(1-\gamma_5)}{2}\begin{pmatrix} p \\ n \end{pmatrix} \qquad T_L = 0,\ T_R = 1/2,\ Y = 1/2$$

Scalar mesons are added to break the symmetry and to give masses to various particles through their nonzero vacuum expectation value. A complex doublet ϕ gives mass to the leptons:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}; \quad <\phi> = \frac{1}{\sqrt{2}}\begin{pmatrix} o \\ v' \end{pmatrix} \qquad T_L = 1/2,\ T_R = 0,\ Y = +1/2$$

A real quartet H gives mass to the hadrons

$$H = \begin{pmatrix} \frac{\sigma+i\pi^o}{\sqrt{2}} & i\pi^+ \\ i\pi^- & \frac{\sigma-i\pi^o}{\sqrt{2}} \end{pmatrix}; \quad <H> = \frac{1}{\sqrt{2}}\begin{pmatrix} v & o \\ o & v \end{pmatrix} \quad T_L = 1/2,\ T_R = 1/2,\ Y = 0$$

The notation for the H fields is chosen to be suggestive. The interaction of H with nucleons is precisely that of the σ-model. Actually, a "physical" pion triplet will emerge later as that linear combination of all the Higgs fields which is an eigenvector of the mass operator. For later purposes, we also include a doublet ρ:

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^o \end{pmatrix}; \quad <\rho> = \frac{1}{\sqrt{2}}\begin{pmatrix} o \\ v'' \end{pmatrix} \qquad T_L = 0,\ T_R = 1/2,\ Y = +1/2$$

By group transformation, we may simultaneously choose <H> proportional to the unit matrix with v real and $<\phi> = \frac{1}{\sqrt{2}}\begin{pmatrix} o \\ v' \end{pmatrix}$ with v' real. However, we are not free to choose the phase of v'' concurrently.

The most general Lagrangian constructed from these fields is

$$\mathcal{L} = -\frac{1}{4}F^{\alpha}_{L\mu\nu}F_L^{\alpha\mu\nu} - \frac{1}{4}F^{\alpha}_{R\mu\nu}F_R^{\alpha\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$- \frac{1}{2}D_\mu\phi^\dagger D^\mu\phi - \frac{1}{2}D_\mu\rho^\dagger D^\mu\rho - \frac{1}{2}D_\mu H^\dagger D^\mu H - V(\phi,\rho,H)$$

$$- \bar{L}\gamma^\mu D_\mu L - \bar{R}\gamma^\mu D_\mu R - \bar{N}_L\gamma^\mu D_\mu N_L - \bar{N}_R\gamma^\mu D_\mu N_R \qquad (1)$$

$$-g_e(\bar{L}\phi R + \text{h.c.}) - g_\pi(\bar{N}_L H N_R + \text{h.c.})$$

where

a) $$F^{\alpha}_{L\mu\nu} = \partial_\mu A^{\alpha}_{L\nu} - \partial_\nu A^{\alpha}_{L\mu} - g_L\varepsilon^{\alpha\beta\gamma}A^{\beta}_{L\mu}A^{\gamma}_{L\nu}$$

$$F^{\alpha}_{R\mu\nu} = \partial_\mu A^{\alpha}_{R\nu} - \partial_\nu A^{\alpha}_{R\mu} - g_R\varepsilon^{\alpha\beta\gamma}A^{\beta}_{R\mu}A^{\gamma}_{R\nu}$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

g_L is the coupling constant associated with $SU(2)_L$; g_R with $SU(2)_R$; and g_Y with $U(1)$. They are assumed to be small (of order e), whereas g_π, the Yukawa coupling between the "pions" and nucleons, may be large.

b) The covariant derivatives of the scalar and spinor fields are

$$D_\mu\phi = (\partial_\nu + ig_L\frac{\vec{\tau}}{2}\cdot\vec{A}^L_\mu + \frac{ig_Y B_\mu}{2})\phi$$

$$D_\mu\rho = (\partial_\mu + ig_R\frac{\vec{\tau}}{2}\cdot\vec{A}^R_\mu + \frac{ig_Y B_\mu}{2})\rho$$

$$D_\mu H = (\partial_\mu H + ig_L\frac{\vec{\tau}}{2}\cdot\vec{A}^L_\mu H - ig_R H\frac{\vec{\tau}}{2}\cdot\vec{A}^R_\mu)$$

$$D_\mu L = (\partial_\mu + ig_L\frac{\vec{\tau}}{2}\cdot\vec{A}^L_\mu - \frac{ig_Y B_\mu}{2})L$$

$$D_\mu R = (\partial_\mu - ig_Y B_\mu)R$$

$$D_\mu N_L = (\partial_\mu + ig_L\frac{\vec{\tau}}{2}\cdot\vec{A}^L_\mu + \frac{ig_Y B_\mu}{2})N_L$$

$$D_\mu N_R = (\partial_\mu + ig_R \frac{\vec{\tau}}{2} \cdot \vec{A}^R_\mu + \frac{ig_Y B_\mu}{2}) N_R$$

c) $V(\phi, \rho, H)$ is the most general gauge invariant fourth order polynomial in the fields:

$$V(\phi,\rho,H) = a\rho^\dagger\rho + b(\rho^\dagger\rho)^2 + c(\phi^\dagger\phi) + d(\phi^\dagger\phi)^2 + e(H^\dagger H) + f(H^\dagger H)^2 +$$

$$+ h(\phi^\dagger H\rho + \rho^\dagger H^\dagger\phi) + j(\rho^\dagger\rho)(\phi^\dagger\phi) + k(\rho^\dagger\rho)(H^\dagger H) + l(\phi^\dagger\phi)(H^\dagger H) \qquad (2)$$

The condition that the potential be a classical minimum: $\langle\frac{\partial V}{\partial\psi}\rangle_{VEV} = 0$, where ψ is any of the scalar fields, gives relations for the vacuum expectation values:

$$2v'(c + dv'^2 + lv^2 + j\frac{|v''|^2}{2}) + \sqrt{2}hvv''^* = 0$$

$$2v''^*(a + b|v''|^2 + kv^2 + j\frac{v'^2}{2}) + \sqrt{2}hvv' = 0 \qquad (3)$$

$$v(2e + 4fv^2 + k|v''|^2 + lv'^2) + \frac{hv'}{2\sqrt{2}}(v'' + v''^*) = 0$$

$$hv'(v'' - v''^*) = 0$$

These equations must be satisfied for a range of coupling constants a,b,c,...l if the theory is to be renormalizable. Therefore unless $h \equiv 0$, which can be achieved by imposing an added discrete symmetry on the Lagrangian, then v'' is also real.

To calculate physical processes, we shift the scalar fields and define new fields which have no vacuum expectation value:

$$\phi = \phi' + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v' \end{pmatrix}, \qquad \langle\phi'\rangle = 0, \text{ etc.}$$

After shifting fields, all but one of the previously massless vector mesons as well as the electron and the nucleons acquire mass. Six of the scalars, the Goldstone bosons, have zero mass while the remaining scalars pick up mass.

The zeroth order vector meson mass matrix becomes

$$\mu^2 = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & B \end{pmatrix}$$ where A is the mass matrix for the charged sectors and B for the three neutral vector mesons.

$$A = \frac{1}{8}\begin{pmatrix} g_L^2(v'^2+2v^2) & -2g_Lg_Rv^2 \\ -2g_Lg_Rv^2 & g_R^2(v''^2+2v^2) \end{pmatrix}$$

$$B = \frac{1}{8}\begin{pmatrix} g_L^2\ (v'^2+2v^2) & -2g_Lg_Rv^2 & -g_Yg_Lv'^2 \\ -2g_Lg_Rv^2 & g_R^2(v''^2+2v^2) & -g_Yg_Rv''^2 \\ -g_Yg_Lv'^2 & -g_Yg_Rv''^2 & g_Y^2(v'^2+v''^2) \end{pmatrix}$$

The mass eigenvalues are

$$\mu_{A\pm}^2 = \frac{1}{16}[g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2)\pm\{(g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2))^2- 4g_L^2g_R^2(v'^2v''^2+2v^2v'^2+2v^2v''^2)\}^{1/2}]$$

$$\mu_{B\pm}^2 = \frac{1}{16}[g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2)+g_Y^2(v'^2+v''^2)+ \pm\{(g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2)+g_Y^2(v'^2+v''^2))^2-4(g_L^2g_R^2+g_L^2g_Y^2+ g_R^2g_Y^2)(v'^2v''^2+2v^2v'^2+2v^2v''^2)\}^{1/2}]$$

The massless photon A_μ corresponds to the linear combination

$$A_\mu = \frac{g_Rg_YA_\mu^{L3}+g_Lg_YA_\mu^{R3}+g_Lg_RB_\mu}{(g_L^2g_R^2+g_L^2g_Y^2+g_R^2g_Y^2)^{1/2}}$$

The electric charge is

$$e = g_L g_R g_Y / \left(g_L^2 g_R^2 + g_L^2 g_Y^2 + g_R^2 g_Y^2\right)^{1/2}$$

The electron picks up a mass $m_e = g_e v'/\sqrt{2}$. The proton and neutron have the same zeroth order mass $m = m_p = m_n = g_\pi v$. Therefore the proton-neutron mass difference is finite and calculable in the model. We will return to this in section III.

The scalar mass matrix is given by

$$M^2_{ij} = \left\langle \frac{\partial^2 V(\phi, \rho, H)}{\partial\psi_i \partial\psi_j} \right\rangle$$

The condition that the potential be a true minimum is equivalent to the condition that the scalar mass matrix be semi-positive definite. Six linear combinations of scalars (2 positively charged, 2 negatively charged, 2 neutral) have zero mass to all order. These are the Goldstone bosons which get "eaten up" to become the longitudinal component of the six massive vector mesons. There remain three massive neutrals and a mass degenerate triplet with charges +, 0, - and with mass

$$m^2 = -\frac{h}{\sqrt{2}}\left(\frac{vv'}{v''} + \frac{vv''}{v'} + \frac{v'v''}{v}\right) \tag{4}$$

The triplet of scalars, which we shall interpret as pions, is

$$\Pi^o = \frac{v'v''\pi^o - vv'\left(\frac{\rho^o - \bar{\rho}^o}{\sqrt{2}i}\right) + vv''\left(\frac{\phi^o - \bar{\phi}^o}{\sqrt{2}i}\right)}{\left(v^2 v'^2 + v^2 v''^2 + v'^2 v''^2\right)^{1/2}} \tag{5}$$

$$\Pi^+ = \frac{v'v''\pi^+ - ivv'\rho^+ + ivv''\phi^+}{\left(v^2 v'^2 + v^2 v''^2 + v'^2 v''^2\right)^{1/2}}, \quad \Pi^- = (\Pi^+)^\dagger$$

Since we have a zero order mass relation, the pion mass difference is also finite and calculable in the model (see section IV). The remaining massive neutral scalars are linear combinations of $\frac{\rho^0+\bar{\rho}^0}{\sqrt{2}}$, $\frac{\phi^0+\bar{\phi}^0}{\sqrt{2}}$, σ with mass matrix

$$M^2 = \begin{bmatrix} 2bv''^2 - \frac{hvv'}{\sqrt{2}v''} & jv'v'' + \frac{hv}{\sqrt{2}} & 2kvv'' + \frac{hv'}{\sqrt{2}} \\ jv'v'' + \frac{hv}{\sqrt{2}} & 2dv'^2 - \frac{hvv''}{\sqrt{2}v'} & 2lvv' + \frac{hv''}{\sqrt{2}} \\ 2kvv'' + \frac{hv'}{\sqrt{2}} & 2lvv' + \frac{hv''}{\sqrt{2}} & 8fv^2 - \frac{hv'v''}{\sqrt{2}v} \end{bmatrix}$$

Before turning to the nucleon-pion sector which is the main concern of this paper, we will require that the model incorporate universality of the weak interactions. The pieces of the Lagrangian which contribute to β decay and μ decay are

$$\mathcal{L}_\mu = G_\mu[\bar{e}\gamma^\mu(\frac{1+\gamma_5}{2})\nu_e][\bar{\nu}_\mu\gamma_\mu(\frac{1+\gamma_5}{2})\mu]$$

$$\mathcal{L}_\beta = G_\beta[\bar{e}\gamma^\mu\frac{(1+\gamma_5)}{2}\nu_e][\bar{p}\gamma_\mu\frac{(1+A\gamma_5)}{2}n]$$

Weak interaction universality in gauge models means that $G_\mu \approx G_\beta$ and $A \approx 1$ in the approximation that vector meson momenta in propagators are ignored relative to vector meson masses. In our model

$$\frac{G_\beta - G_\mu}{G_\mu} = \frac{2v^2}{v''^2+2v^2} \qquad A = \frac{v''^2}{v''^2+4v^2}$$

Both conditions are satisfied for $v \ll v''$. Therefore our implementation of universality is unnatural in the technical sense that it depends on one parameter in the theory being much smaller than another.

III. The Proton Neutron Mass Difference

In this section we calculate the proton-neutron mass difference in several stages. First we calculate $\Delta m|_{p-n}$ for the Higgs content specified in the previous section. Then we examine the dependence of Δm on the Higgs content for any general irreducible Higgs multiplet. We then exhibit a modification of the Higgs system of section II which gives the correct sign, $\Delta m|_{p-n} < 0$, and also maintains the other features of the model (massive electron, equal nucleon masses, etc.) discussed above.

A. Higgs content of section II

The calculation with fixed Higgs content (ϕ,H) has already been made by others.[5] For this content, it was found that $\Delta m = m_p - m_n > 0$, which is the wrong sign at least from the point of view of the naive quark model. The sign remains positive when we add the ρ Higgs field.

In the one loop calculation of the mass difference, there are three types of diagrams that may contribute--vector meson exchange, scalar meson exchange and tadpole diagrams (Figure 2). In this problem, the tadpoles contribute equally to the proton neutron self masses, since contributions proportional to γ_5 only contribute to wave function renormalization in this order. If we neglect the nucleon mass relative to the vector meson masses, then we can ignore the scalar meson exchange term. In fact, in this model it is identically zero. This leaves vector meson exchange. The contribution from the $k_\mu k_\nu$ part of the vector meson propagator vanishes when the fermions are on mass shell and therefore does not contribute to Δm. Thus we find

$$\Delta m = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\gamma_\mu \frac{(-i(\not{p}-\not{k})+m)}{(p-k)^2+m^2}\gamma^\mu [g_Y g_L (k^2+\mu^2)^{-1}_{L3,B} + g_Y g_R (k^2+\mu^2)^{-1}_{R3,B}] \tag{6}$$

This formula is more general than for the specific Higgs content stated above. The quantity in square brackets has the general form

$$\frac{A+Bk^2}{k^2(k^2+\mu_1^2)(k^2+\mu_2^2)} \tag{7}$$

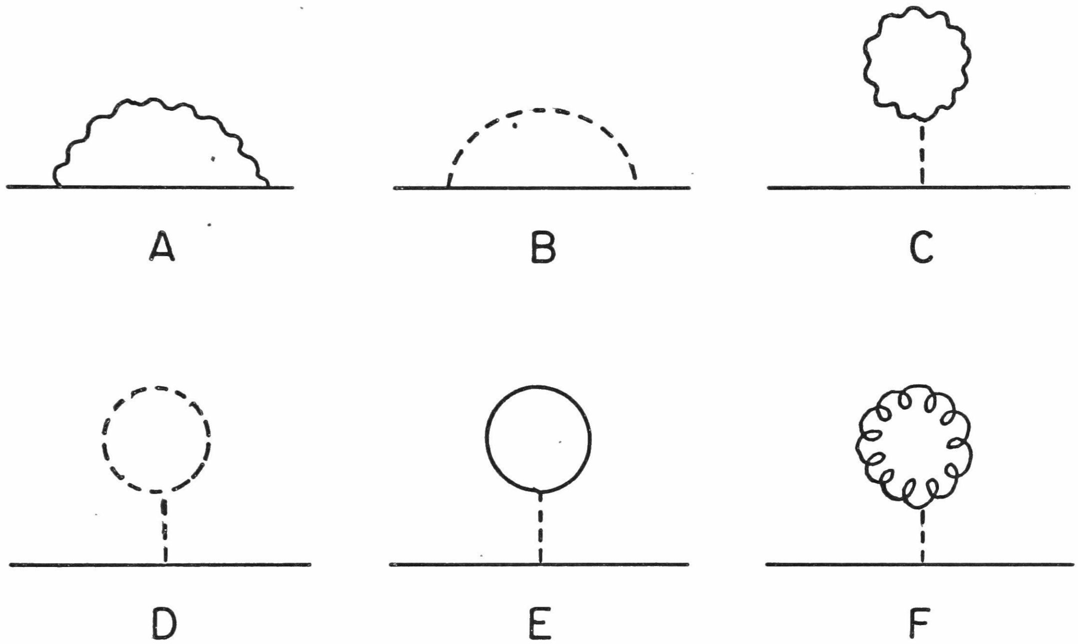

Figure 2. Diagrams which contribute to the fermion self-mass in the one loop approximation: A. vector meson exchange, B. Higgs scalar exchange, C. vector meson tadpole, D. Higgs tadpole, E. fermion tadpole, F. ghost tadpole

where μ_1 and μ_2 are the neutral vector meson mass eigenvalues. Simplifying the integral, we find

$$\Delta m = -\frac{m\pi^2}{(2\pi)^4} \frac{1}{\mu_1^2-\mu_2^2}\left\{\frac{A}{m^2}\left[J\frac{(\mu_1^2/m^2)}{\mu_1^2/m^2} - J\frac{(\mu_2^2/m^2)}{\mu_2^2/m^2}\right]-B\left[J(\mu_1^2/m^2)-J(\mu_2^2/m^2)\right]\right\} \tag{8}$$

$$J(\beta) = \int_0^1 dx(1+x)\ln\left(1+\frac{1-x}{x^2}\beta\right)$$

Pi has shown that $J(\beta)$ is a monotonically increasing function of β, whereas $J(\beta)/\beta$ is monotonically decreasing.[5] Therefore the conditions $A\leq 0$, $B\leq 0$ together are sufficient to guarantee the correct sign for Δm.

For the Higgs content of Section II, we find that

$$g_L g_Y (k^2+\mu^2)^{-1}_{L3,B} = \frac{g_L^2 g_Y^2}{8k^2(k^2+\mu_1^2)(k^2+\mu_2^2)}\left[v'^2k^2+\frac{g_R^2}{8}(v'^2v''^2+2v^2v'^2+2v^2v''^2)\right]$$

$$g_R g_Y (k^2+\mu^2)^{-1}_{R3,B} = \frac{g_R^2 g_Y^2}{8k^2(k^2+\mu_1^2)(k^2+\mu_2^2)}\left[v''^2k^2+\frac{g_L^2}{8}(v'^2v''^2+2v^2v'^2+2v^2v''^2)\right]$$

in which case $A>0$ and $B>0$, so that we find the wrong sign for Δm. This calculation contains the earlier result (without the ρ field) by setting $v''=0$.

B. One Irreducible Higgs Representation

We next ask the question: is the sign of Δm typical for this gauge group or does it depend on the representation content chosen and vary with the way the symmetry is broken? Since the only kind of symmetry breaking we know how to calculate is via the Higgs mechanism, we look at the sign of Δm for general Higgs content. Equations (6)-(8) for the mass difference are valid for any Higgs system, except that A and B depend on the Higgs sector through the neutral vector meson mass matrix:

$$A = g_L g_Y(\mu^2_{L3,R3}\mu^2_{R3,B}-\mu^2_{L3,B}\mu^2_{R3,R3})+g_R g_Y(\mu^2_{L3,R3}\mu^2_{L3,B}-\mu^2_{R3,B}\mu^2_{L3,L3})$$

$$B = -g_L g_Y\mu^2_{L3,B}-g_R g_Y\mu^2_{R3,B} \tag{9}$$

Condition $B \leq 0$ is linear in μ^2 and therefore additive with respect to different Higgs multiplets, whereas Condition $A \leq 0$ is not. Therefore, for simplicity we suppose that the Higgses all belong to a single irreducible representation K_{ij} of the gauge group with quantum numbers $T_L = m$, $T_R = n$, Y. Then

$$T^{ij}_{L3} = \delta_{ij} T^{L3}_{i} = (m-i+1)\delta_{ij}, \quad i = 1,2,\ldots,2m+1$$

$$T^{ij}_{R3} = \delta_{ij} T^{R3}_{i} = (n-i+1)\delta_{ij}, \quad i = 1,2,\ldots,2n+1$$

The contribution of K_{ij} to the neutral vector masses comes from the term in the Lagrangian

$$L_{\substack{\text{neutral}\\ \text{v.m. masses}}} = -\frac{1}{2}\left| g_L T^{L3}_{ik} \langle K_{kj}\rangle A^{L3}_{\mu} - g_R \langle K_{ik}\rangle T^{R3}_{kj} A^{R3}_{\mu} + g_Y Y \langle K_{ij}\rangle B_{\mu}\right|^2$$

$$= -\frac{1}{2}\left| g_L (m-i+1)\langle K_{ij}\rangle A^{L3}_{\mu} - g_R (n-j+1)\langle K_{ij}\rangle A^{R3}_{\mu} + g_Y Y \langle K_{ij}\rangle B_{\mu}\right|^2$$

Since $\langle K_{ij}\rangle = 0$ unless K_{ij} is neutral, we use the relation $Q=T_{L3}+T_{R3}+Y$ to simplify the mass term. $QK_{ij} = (n+m+Y+2-i-j)\, K_{ij}$. Eliminating $j=n+m+Y+2-i$, we define

$$K_i \equiv \left|\langle K_{i,\, n+m+Y+2-i}\rangle\right| \tag{10}$$

Then

$$L_{\substack{\text{neutral}\\ \text{vector meson}\\ \text{masses}}} = -\frac{1}{2}\left| (g_L (m-i+1) A^{L3}_{\mu} - g_R (i-m-Y-1) A^{R3}_{\mu} + g_Y Y B_{\mu}) K_i \right|^2$$

Using this equation to substitute the values of μ^2_{ij} into equation (9) for A, we find that $A \equiv 0$ for an irreducible representation. That leaves condition B which becomes

$$[Y^2 g_R^2 + Y(m-i+1)(g_R^2+g_L^2)]K_i^2 > 0 \tag{11}$$

It is not difficult to satisfy this equation. Suppose for instance we add a Higgs with quantum numbers $T_L=1$, $T_R = 1/2$, $Y = 1/2$. Then B becomes

$$g_R^2K_2^2 - (g_R^2+2g_L^2)K_3^2 > 0$$

This is positive if K_2 is chosen sufficiently larger than K_3.

C. Modification of Section II Higgs Content to give $\Delta m<0$

The model of section II can be modified by adding the scalar $K(T_L=1, T_R=1/2, Y=1/2)$ to the Higgs sector. The other Higgses remain to give mass to the fermions. However, if K_2 is sufficiently large compared to the other vacuum expectation values, Δm has the correct sign. In that case, we also preserve weak universality. Moreover, since K does not couple to the fermions in the theory, it does not alter the zeroth order fermion mass relations. However, we see that it has a profound influence on dynamics via the change it introduces into the vector meson mass matrix. It will also change the physical interpretation of scalars in the Higgs sector since we must include in the potential a term $(K^+K)\cdot(\psi^+\psi)$ for all other Higgs representations ψ.

The point of the calculation is not so much that we should add this particular Higgs K to our model, for criteria of economy in model building make us reluctant to do so. More importantly, the lesson we learn is that by changing the Higgs content in a theory, even when the new scalars do not interact directly with fermions, we can radically alter the calculable masses and mass differences.

IV. Pions

A. Massive Pions

Returning to the model of section II, we recall that we have a degenerate triplet of pions with zeroth order mass given by equation (4). Therefore to insure $m_\pi^2 \sim 0$, the trilinear Higgs coupling constant h would have to be small of order $h \sim em_\pi^2/\mu$. Although this implementation of the model can give a realistic pion mass, we think that an acceptable model of pions should explain why they are so light. Since we have a zeroth order mass relation, we can calculate the mass difference $\delta m^2 = m_{\pi+}^2 - m_{\pi o}^2$.

The diagrams which contribute to the pion self energy are shown in Figure 3. We expect them to be of order either αm_π^2 or $\alpha\mu^2$. Since there are so many diagrams to calculate, we would like to be able to pick out the $\alpha\mu^2$ diagrams and ignore the αm_π^2 diagrams. In part B we will discover a trick to enable us to do this and therefore we'll complete the calculation of δm^2 in part C.

B. PseudoGoldstone Boson Realization

We now look more closely at the potential in the Lagrangian [equation (2)]. The invariance group of V coincides with the gauge group, SU(2) x SU(2) x U(1). However, if we demand that the Lagrangian be invariant under a reflection symmetry R, which sends ρ into $-\rho$ but sends all other fields into themselves, then the trilinear term $h(\phi^+ H\rho + \rho^+ H^+ \phi)$ drops from the potential. In that case V is invariant under a larger symmetry group. If we rewrite the fields in terms of real fields,

$$P=\frac{\rho^o+\bar{\rho}^o}{\sqrt{2}} \qquad P'=\frac{\rho^o-\bar{\rho}^o}{\sqrt{2}i} \qquad S=\frac{\rho^++\rho^-}{\sqrt{2}} \qquad S'=\frac{\rho^+-\rho^-}{\sqrt{2}i}$$

$$F=\frac{\phi^o+\bar{\phi}^o}{\sqrt{2}} \qquad F'=\frac{\phi^o-\bar{\phi}^o}{\sqrt{2}i} \qquad T=\frac{\phi^++\phi^-}{\sqrt{2}} \qquad T'=\frac{\phi^+-\phi^-}{\sqrt{2}i} \tag{12}$$

$$\sigma \qquad \pi^o \qquad U=\frac{\pi^++\pi^-}{\sqrt{2}} \qquad U'=\frac{\pi^+-\pi^-}{\sqrt{2}i}$$

Then each of the Higgs fields ϕ, H, ρ is a real 4-vector and the potential is a function only of their lengths

$$V(\phi,\rho,H)=V(P^2+P'^2+S^2+S'^2,\ F^2+F'^2+T^2+T'^2,\ \sigma^2+\pi^{o2}+U^2+U'^2)$$

Thus V is invariant under the larger group $\bar{G}$=O(4) x O(4) x O(4). Each 4-vector picks a direction when it acquires nonzero vacuum expectation value and therefore $\bar{S}$=O(3) x O(3) x O(3). Counting dimensions, since $d(\bar{G})=18$, $d(\bar{S})=9$ and $G\cap\bar{S}=U(1)$, we find that

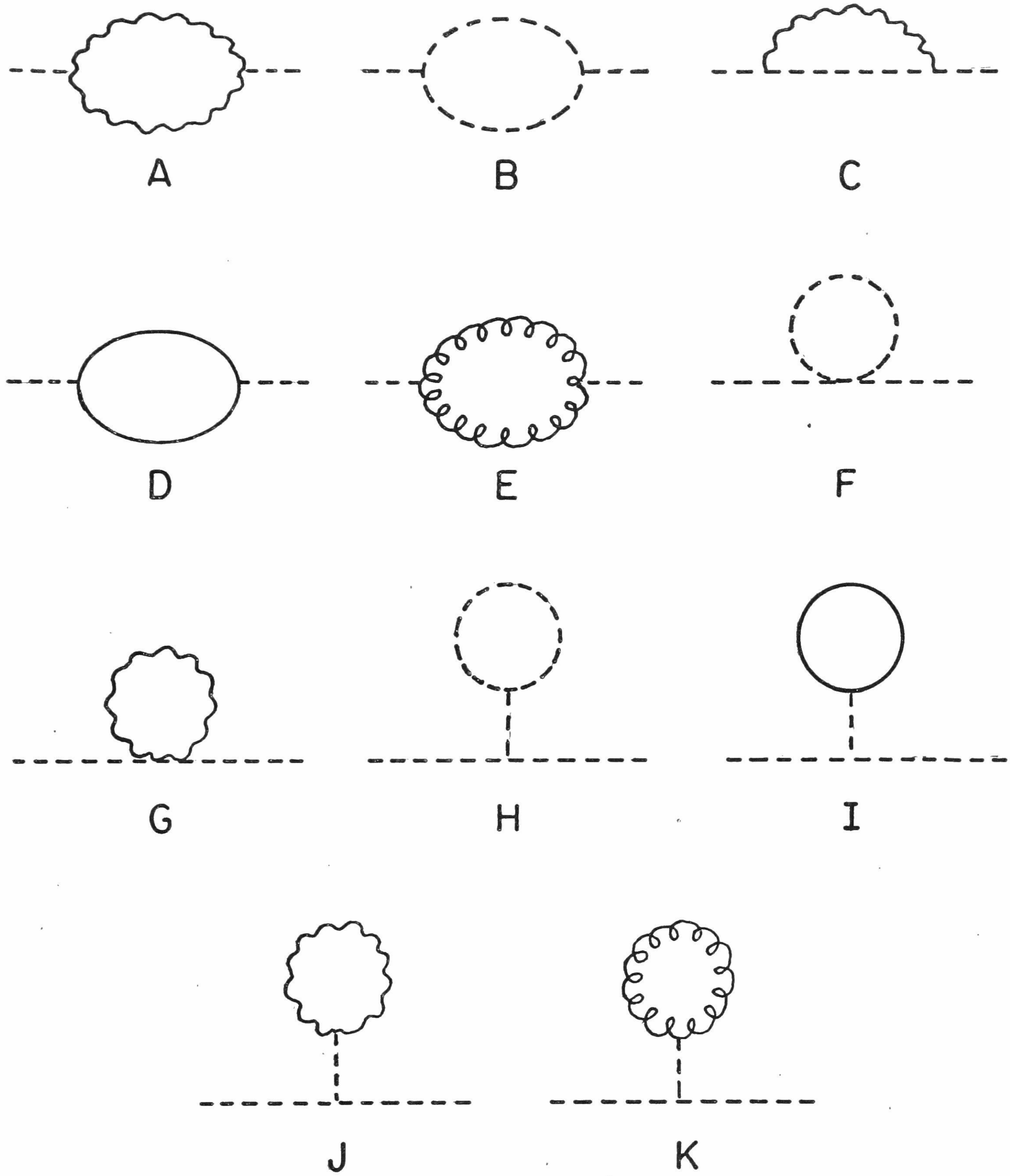

Figure 3. Diagrams which contribute to scalar self-mass in the one loop approximation: A. vector meson exchange, B. scalar exchange, C. vector meson-scalar exchange, D. fermion exchange, E. ghost exchange, F. scalar seagull, G. vector meson seagull, H. scalar tadpole, I. fermion tadpole, J. vector meson tadpole, K. ghost tadpole.

$$\text{number of pseudoGoldstone bosons} = d(\bar{G}-\bar{S}UG)=d(\bar{G})-(d(\bar{S})+d(G)-d(\bar{S}\cap G))$$
$$= 18-(9+7-1)=3$$

Thus we have precisely three pseudoGoldstone bosons which we are going to connote as pions. They are the Π triplet of section II. Their zero order mass, $m^2 = \frac{-h}{\sqrt{2}}\left(\frac{vv'}{v''} + \frac{vv''}{v'} + \frac{v'v''}{v}\right)$, becomes zero when the reflection symmetry R is imposed ($h\equiv 0$).

Thus we have two options to choose from: our model of section II with or without symmetry R. In the first case we can calculate the pion masses and expect them to be small; in the second the triplet mass is arbitrary and there is no reason for it to be small, but we can calculate δm^2.

In the pseudoGoldstone boson realization, we calculate the pion mass by examining the eleven diagrams of Figure 3. Weinberg has shown that a number of cancellations occur when we specialize to the pseudoGoldstone self masses and that the result is ξ independent. Furthermore, the fermion diagrams (D and I) do not contribute when the Yukawa interactions are invariant under $\bar{G}$, as is the case in our model. The only diagrams which contribute are A, G and J with the ξ independent piece of the vector meson propagator instead of the total propagator used in the calculation.[8] The necessary Higgs-vector meson vertices are given in the Appendix.

For the neutral pion mass, diagram A is absent because the Π^o does not couple to two vector mesons. Moreover, diagrams G and J exactly cancel so that $m_{\pi o}^2=0$ to first order as well. Using the relation (5), we write

$$G(\Pi^o,\Pi^o) = \frac{1}{v^2v'^2+v^2v''^2+v'^2v''^2}[v'^2v''^2G(\pi^o,\pi^o)+v^2v'^2G(P',P')+v^2v''^2G(F',F')]$$

$$J(\Pi^o,\Pi^o) = \frac{1}{v^2v'^2+v^2v''^2+v'^2v''^2}[v'^2v''^2J(\pi^o,\pi^o)+v^2v'^2J(P',P')+v^2v''^2J(F',F')]$$

For simplicity we shall assume that the coupling constants j,k,l in V (see eq. (2)) are absent--this makes the F,P,σ mass matrix diagonal. Looking at the F' contribution to G and J we find

$$G(F',F') = -\frac{1}{16}(3x2)\int\frac{d^4k}{(2\pi)^4}\{g_L^{\ 2}[(k^2+\mu^2)^{-1}_{L1,L1}+(k^2+\mu^2)^{-1}_{L2,L2}+(k^2+\mu^2)^{-1}_{L3,L3}]+$$

$$g_Y^{\ 2}(k^2+\mu^2)^{-1}_{B,B}-2g_Lg_Y(k^2+\mu^2)^{-1}_{L3,B}\}$$

$$J(F',F') = -\frac{g_{F'F'F}v'}{m_F^2\ 8}(3x2)\int\frac{d^4k}{(2\pi)^4}\{g_L^{\ 2}[(k^2+\mu^2)^{-1}_{L1,L1}+(k^2+\mu^2)^{-1}_{L2,L2}+$$

$$(k^2+\mu^2)^{-1}_{L3,L3}]+g_Y^{\ 2}(k^2+\mu^2)^{-1}_{B,B}-2g_Lg_Y(k^2+\mu^2)^{-1}_{L3,B}\} \qquad (13)$$

The factor 3 comes from the ξ independent piece of $g^{\mu\nu}\Delta^A_{\alpha\mu,\beta\nu}(k)$ = $\left(\frac{3}{k^2+\mu^2}+\frac{1}{\xi k^2+\mu^2}\right)_{\alpha\beta}$ and the factor 2 comes from counting in the vector meson loop. Since $g_{F'F'F} = -dv'$ and $m_F^2 = 2dv'^2$, the two diagrams cancel. Similarly, the Π^o and P', G and J diagrams cancel. For general j,k,l, equations (13) are modified, but the calculation gives the same result-- $m_{\pi o}^2 \equiv 0$ in the one loop approximation.

When we calculate $m_{\pi+}^2$, diagram A is no longer zero. Moreover, there are cross terms which contribute to the physical Π mass from the A diagram. For example, consider

$$\Pi^1 = \frac{\Pi^+ + \Pi^-}{\sqrt{2}} = \frac{1}{v^2v'^2+v^2v''^2+v'^2v''^2}(v'v''U + vv'S'-vv''T')$$

Then we have

$$A(\Pi^1,\Pi^1) = \frac{1}{v^2v'^2+v^2v''^2+v'^2v''^2}[v'^2v''^2A(U,U)+v^2v'^2A(S',S')+v^2v''^2A(T',T')+$$

$$2vv'^2v''A(U,S')-2vv'v''^2A(U,T')-2v^2v'v''A(S',T')]$$

For example,

$$A(T',T') = \tfrac{3}{16}v'^2g_L^{\ 2}g_Y^{\ 2}\int\frac{d^4k}{(2\pi)^4}(k^2+\mu^2)^{-1}_{L1,L1}(k^2+\mu^2)^{-1}_{B,B}$$

Similarly in the approximation in which we set j,k,l = 0,

$$G(T',T') = -\frac{3x2}{16}\int\frac{d^4k}{(2\pi)^4}\{g_L^{\ 2}[(k^2+\mu^2)^{-1}_{L1,L1}+(k^2+\mu^2)^{-1}_{L2,L2}+(k^2+\mu^2)^{-1}_{L3,L3}] +$$

$$g_Y^{\ 2}(k^2+\mu^2)^{-1}_{B,B}+2g_Lg_Y(k^2+\mu^2)^{-1}_{L3,B}\}$$

$$J(T',T') = \frac{3\times 2}{16}\int\frac{d^4k}{(2\pi)^4}\{g_L{}^2[(k^2+\mu^2)^{-1}_{L1,L1}+(k^2+\mu^2)^{-1}_{L2,L2}+(k^2+\mu^2)^{-1}_{L3,L3})+$$

$$g_Y{}^2(k^2+\mu^2)^{-1}_{B,B}-2g_Lg_Y(k^2+\mu^2)^{-1}_{L3,B}\}$$

Substituting in these equations the values of the inverse propagator matrix we find

$$\Sigma(T',T') = A(T',T') + G(T',T') + J(T',T')$$

$$= -\frac{3}{2}\, g_Lg_Y\int\frac{d^4k}{(2\pi)^4}\,[(k^2+\mu^2)^{-1}_{L3,B}\frac{g_Lg_Yv'^2}{8}(k^2+\mu^2)^{-1}_{L1,L1}(k^2+\mu^2)^{-1}_{B,B}]$$

$$= -\frac{3}{2}\,\frac{g_L{}^2g_R{}^2g_Y{}^2v^2v''^2}{32}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2(k^2+\mu_1^2)(k^2+\mu_2^2)}$$

$$= -\frac{3}{64}\,g_L{}^2g_R{}^2g_Y{}^2v^2v''^2\,(\frac{i\pi^2}{(2\pi)^4})\,\frac{\log(\mu_1^2/\mu_2^2)}{\mu_1^2-\mu_2^2}$$

$$m^2_{T',T'} = i\Sigma(T',T') = \frac{3\pi^2}{64(2\pi)^4}\,g_L{}^2g_R{}^2g_Y{}^2v^2v''^2\,\frac{\log(\mu_1^2/\mu_2^2)}{\mu_1^2-\mu_2^2}$$

In a similar way, we can compute the remaining matrix elements of $m^2_{ij}(i,j=S',T',U)$

$$m^2 = \begin{pmatrix} v^2v'^2 & -v'v''v^2 & -vv'^2v'' \\ -v'v''v^2 & v^2v''^2 & vv'v''^2 \\ -vv'^2v'' & vv'v''^2 & v'^2v''^2 \end{pmatrix} \times \frac{3\pi^2}{64(2\pi)^4}\,g_L{}^2g_R{}^2g_Y{}^2\,\frac{\log(\mu_1^2/\mu_2^2)}{\mu_1^2-\mu_2^2}$$

We note that m^2 is proportional to the zero order mass matrix when $h\neq 0$— namely the linear combinations of Higgs scalars which were true Goldstone bosons pick up no mass in one loop (and to all higher orders as well). The charged pions acquire mass

$$m^2_{\pi+} = \frac{3\pi^2}{64(2\pi)^4}\,g_L{}^2g_R{}^2g_Y{}^2\,\frac{\log(\mu_1^2/\mu_2^2)}{\mu_1^2-\mu_2^2}\,(v^2v'^2+v^2v''^2+v'^2v''^2)$$

This is of order $\alpha\mu^2$, but perhaps we should not rule it out too soon since by experimenting with different values of the parameters $(g_L, g_R, g_Y, v, v', v'')$ we find that we can damp this by factors of 10^{-2}.

If on the other hand, we wish to interpret this calculation of $m^2_{\pi+}$ as a calculation of δm^2 (since $m^2_{\pi o}=0$) we seem to be in trouble. We would then need damping by at least 10^{-3} which seems highly unlikely in this model. Furthermore, to agree with experiment, the mass of the Π^o, a two loop calculation, would have to be an order of magnitude greater than δm^2, a one loop quantity. This doesn't make much sense.

Thus in our model we find that the mass differences of pseudoGoldstone bosons are of the same order as the pseudoGoldstone boson masses. We expect that this may be a general feature of models which implement the pseudoGoldstone mechanism in weak and electromagnetic gauge theories.

C. δm^2 for Massive Pions

We now return to the calculation of δm^2 in the version of the model with massive zeroth order pions. We can apply Weinberg's analysis of the scalar self-mass in the pseudoGoldstone boson case to eliminate those diagrams of order αm^2_π. They are precisely those diagrams which cancel out when h=0. Since h=0 corresponds to the pseudoGoldstone realization, we can use our calculation of the pseudoGoldstone masses to find the leading contribution to $\delta m^2_{(\mathrm{massive})}$:

$$\delta m^2_{(\mathrm{massive})} \approx (m^2_{\Pi+} - m^2_{\Pi o})_{\mathrm{pseudoGoldstone}}$$

$$= \frac{3\Pi^2}{64(2\Pi)^4}\, g_L^2 g_R^2 g_Y^2 \left(\frac{\log \mu_1^2/\mu_2^2}{\mu_1^2 - \mu_2^2}\right) (v^2v'^2+v^2v''^2+v'^2v''^2)$$

Thus δm^2 is of order $\alpha\mu^2$ and it is unlikely that this can be damped sufficiently to account for the $\Pi^+ - \Pi^o$ mass difference (see Figure 4).

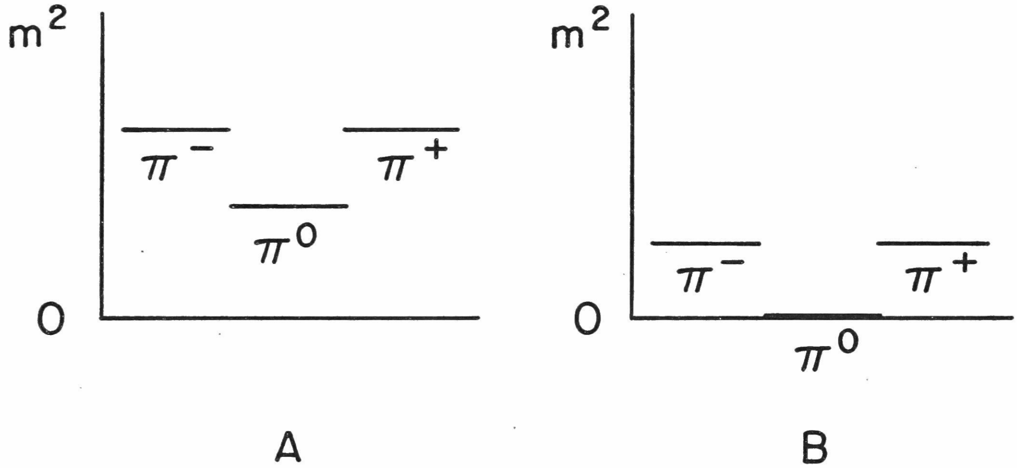

Figure 4. Pion mass spectrum in one loop approximation. A) Presence of zeroth order pion mass (no reflection symmetry R in Lagrangian). B) PseudoGoldstone boson realization (reflection symmetry R).

V. Parity Violation

Gauge models with strongly interacting scalar fields appear to violate parity to order α, which is unacceptably large. In our model we look at this question for the pion-nucleon form factor as a preliminary study. The diagrams which may contribute in the one loop approximation to the proper pion-nucleon vertex are shown in Figure 5; we are interested only in the parity nonconserving piece of each diagram.

We look at the special case $\Pi^o pp$ for simplicity. Diagram A does not contribute to parity violation because of the symmetry of the vector meson coupling. Diagrams C and D do not contribute because the Higgs-fermion system is parity conserving. Diagram E is absent because the Π^o does not couple to two vector mesons. The parity violating piece of the B diagrams is:

$$B_I = -\frac{ig_\pi}{4}\int\frac{d^4 l}{(2\pi)^4}\frac{(2k-l)_\nu}{(k-l)^2+m_\sigma^2}\frac{\gamma_\mu[-i(\not{p}-\not{l})+m]}{(p-l)^2+m^2}\left[-\frac{g_L^2}{2}\Delta^{\mu\nu}_{L3,L3}(l)-g_Lg_Y\Delta^{\mu\nu}_{B,L3}(l)+\frac{g_R^2}{2}\Delta^{\mu\nu}_{R3,R3}(l)+g_Rg_Y\Delta^{\mu\nu}_{B,R3}(l)\right]$$

$$B_{II} = -\frac{ig_\pi}{4}\int\frac{d^4 l}{(2\pi)^4}\frac{(2k-l)_\nu}{(k-l)^2+m_\sigma^2}\frac{[-i(\not{p}'+\not{l})+m]\gamma_\mu}{(p'+l)^2+m^2}\left[-\frac{g_L^2}{2}\Delta^{\mu\nu}_{L3,L3}(l)-g_Lg_Y\Delta^{\mu\nu}_{B,L3}(l)+\frac{g_R^2}{2}\Delta^{\mu\nu}_{R3,R3}(l)+g_Rg_Y\Delta^{\mu\nu}_{B,R3}(l)\right]$$

Taking the sum, in the $\xi=1$ gauge for simplicity and substituting the vector meson masses in the formulas, we find

$$B = \frac{ig_\pi}{4}\int\frac{d^4 l}{(2\pi)^4}\left[l^2\frac{(g_L^2-g_R^2)}{2}+\frac{g_L^2g_R^2}{16}(v'^2-v''^2)-\frac{g_L^2g_Y^2}{16}(3v'^2+v''^2)+\frac{g_R^2g_Y^2}{16}(v'^2+3v''^2)\right.$$
$$\left.\times\left[\frac{((p'+l)^2+m^2)(2\not{k}-\not{l})(i\not{p}-i\not{l}-m)+((p-l)^2+m^2)(i\not{p}'+i\not{l}-m)(2\not{k}-\not{l})}{((k-l)^2+m_\sigma^2)(l^2+\mu_1^2)(l^2+\mu_2^2)((p-l)^2+m^2)((p'+l)^2+m^2)}\right]\right.$$

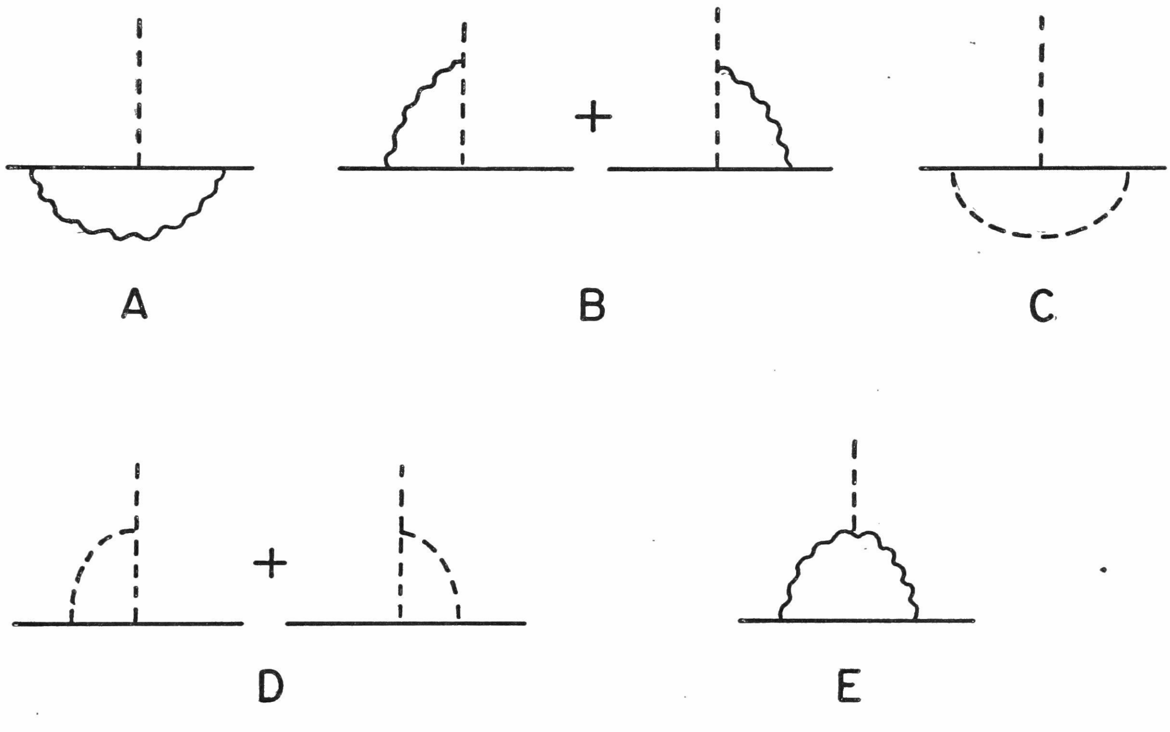

Figure 5. One loop corrections to the pion-nucleon form factor.

The apparent logarithmically divergent piece disappears and the leading contribution behaves like $\alpha(m^2/\mu^2)$ in the region where the external momenta are small compared with the vector meson masses. (We note that $B\equiv 0$ when the left-right symmetry of the model is artifically realized ($g_L=g_R, v'=v''$).) To arrive at this estimate we consider only the leading behavior of the numerator and denominator, which is legitimate since the denominator contains only massive particle propagators and is not plagued by infrared singularities. (A good check on our result is that $B\equiv 0$ when $m=0$.) Thus in the form factor, parity violation is a calculable weak phenomenon. This is certainly fine if we look at the term proportional to g_π, as shown here. However, this is not sufficient since we do not wish to treat g_π as a perturbative constant. However, we expect that our result is more generally valid.

Appendix

In the appendix we give the vertices necessary for the pion mass calculation:

A. Coupling of ϕ with vector mesons

$$\phi A^2 \text{ terms:} \quad -\frac{Fv'}{8}\,[g_L{}^2(A_{L1}^2+A_{L2}^2+A_{L3}^2)+g_Y{}^2B^2-2g_Lg_YA_{L3}B]$$

$$-\frac{Tv'}{4}\,g_Lg_YA_{L1}B+\frac{T'v'}{4}\,g_Lg_YA_{L2}B$$

$$\phi^2A^2 \text{ terms:} \quad -\frac{1}{16}(F^2+F'^2)[g_L{}^2(A_{L1}^2+A_{L2}^2+A_{L3}^2)+g_Y{}^2B^2-2g_Lg_YA_{L3}B]$$

$$-\frac{1}{16}(T^2+T'^2)[g_L{}^2(A_{L1}^2+A_{L2}^2+A_{L3}^2)+g_Y{}^2B^2+2g_Lg_YA_{L3}B]$$

B. Coupling of ρ with vector mesons

$$\rho A^2 \text{ terms:} \quad -\frac{Pv''}{8}[g_R^2(A_{R1}^2+A_{R2}^2+A_{R3}^2)+g_Y{}^2B^2-2g_Rg_YA_{R3}B]$$

$$-\frac{Sv''}{4}\,g_Rg_YA_{R1}B+\frac{S'v''}{4}\,g_Rg_YA_{R2}B$$

$$\rho^2A^2 \text{ terms:} \quad -\frac{1}{16}(P^2+P'^2)[g_R^2(A_{R1}^2+A_{R2}^2+A_{R3}^2)+g_Y{}^2B^2-2g_Rg_YA_{R3}B]$$

$$-\frac{1}{16}(S^2+S'^2)[g_R{}^2(A_{R1}^2+A_{R2}^2+A_{R3}^2)+g_Y{}^2B^2+2g_Rg_YA_{R3}B]$$

C. Coupling of H with vector mesons

$$HA^2 \text{ terms:} \quad -\frac{v\sigma}{4}\,[g_L^2(A_{L1}^2+A_{L2}^2+A_{L3}^2)+g_R{}^2(A_{R1}^2+A_{R2}^2+A_{R3}^2)+2g_Lg_R(A_{L1}A_{R1}+A_{L2}A_{R2}-A_{L3}A_{R3}$$

$$+\frac{vU}{2}\,g_Lg_R(A_{L2}A_{R3}-A_{R2}A_{L3})+\frac{vU'}{2}g_Lg_R(A_{L1}A_{R3}-A_{R1}A_{L3})$$

$$H^2A^2 \text{ terms:} \quad -\frac{(\pi_o^2+\sigma^2)}{8}\,[g_L^2(A_{L1}^2+A_{L2}^2+A_{L3}^2)+g_R^2(A_{R1}^2+A_{R2}^2+A_{R3}^2)+2g_Lg_R(A_{L1}A_{R1}+A_{L2}A_{R2}$$

$$-A_{L3}A_{R3})]-\frac{U^2}{8}[g_L^2(A_{L1}^2+A_{L2}^2+A_{L3}^2)+g_R^2(A_{R1}^2+A_{R2}^2+A_{R3}^2)+2g_Lg_R(-A_{L1}A_{R1}+A_{L2}A_{R2}+A_{L3}A_{R3}$$

$$-\frac{U'^2}{8}[g_L^2(A_{L1}^2+A_{L2}^2+A_{L3}^2)+g_R^2(A_{R1}^2+A_{R2}^2+A_{R3}^2)+2g_Lg_R(A_{L1}A_{R1}-A_{L2}A_{R2}+A_{L3}A_{R3})]$$

Footnotes

1. S. Weinberg, Phys. Rev. D5, 1962 (1972); H. Georgi and S.L. Glashow, Phys. Rev. D6, 2977 (1972).
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