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Reactors of Uniform Power, Fuel Loading, and Flux*

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A tractable mixed numerical-analytic method is given for the approximate design of reflected reactors with cores having simultaneously flat power density, fuel loading, absorber distribution, and moderator distribution. The method as applied to finite cylinders yields a fast digital routine (about 1 sec/calculation on the IBM 704) which gives trustworthy criticality values in systems with important neutron capture at energies above thermal. Corroborative experiments on critical assemblies containing molybdenum and tungsten show power distributions flat to within ±5%.

INTRODUCTION

The problem of designing a flat-flux reactor outdates the first nuclear chain reaction. In 1941 it was observed by Feynman[1] that if uranium is nonuniformly distributed in a uniformly moderated thermal reactor, then flat flux is achieved by the distribution which yields minimum critical mass. Later this fact was rediscovered and elegantly elucidated by Goertzel[2]. The similarly interesting problem of obtaining *uniform power density*, by allowing both fuel loading and flux to be nonuniform, has been analyzed by Goertzel and Loeb[3]. The problem of optimum control poison distribution has been investigated by Lamarsh[4], who showed that an optimum poison distribution again leads to flat flux. Flux flattening by means of nonuniform *moderation* has been investigated by Ravets and Lamarsh[5].

All these investigations are confined to reactors in which substantially all fission is produced by thermal neutrons. More important, in each study one type of uniformity is achieved at the expense of making another engineering parameter change throughout the reactor core, in a fashion that is fairly complicated to specify.

There are evident advantages to a reactor core which is uniform throughout in all design and performance features. The advantages include uniform burnup, uniform fuel-element spacing, uniform metallurgy, interchangeable modular construction, ease of engineering design and assembly, peak performance throughout the entire core. In particular, for a reactor of extreme performance, such a design simplification becomes a very important convenience. In this paper it will be shown how uniform power density may be achieved in conjunction with *entirely uniform core construction*, by means of appropriate reflector design, for a reactor of such a core composition that a significant fraction of neutrons fail to reach thermal energy. In practice the method yields reactors whose reflectors are compatible with good shielding.

The method presented is analytic and feasible for hand calculation, and it may be used as the basis of an exceedingly rapid machine calculation routine yielding the design parameters of critical reactors featuring uniform power, loading, and flux. The essential idea of the method is that reflector thickness must be so chosen that buckling leakage from the reflector just balances capture in the core. Although the method is approximate,

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1R. P. FEYNMAN, (1941), Unpublished.
2G. GOERTZEL, *J. Nucl. Energy*, 2, 193-201 (1956).
3G. GOERTZEL and W. A. LOEB, *Nucleonics*, 12, No. 9, 42-45 (1954).
4J. R. LAMARSH, B. N. L. Memorandum, (1956), Unpublished.
5J. M. RAVETS and J. R. LAMARSH, *Nuclear Sci. and Eng.*, 7, 496-501 (1960).

it has been checked in detail against well-proven reactor codes based on direct numerical integration, and has been in good agreement. As an ultimate check, a large number of low-power critical assembly experiments have been designed by use of the method. Fission profiles have proven to be flat to typically better than ±5%, while excellent precision has been obtained in predicting critical masses.

The moderator and reflector materials for which the following development is applicable are hydrogen, deuterium, beryllium, and carbon, all of which have scattering cross-sections approximately of the form

$$\sigma_m(u) = \sigma_{m\,th}\, f(u) \tag{1}$$

where u is the lethargy;
m indexes the material;
$\sigma_{m\,th}$ is the cross-section at thermal lethargy;
and $f(u)$ is a universal function with value 1 at thermal lethargy and independent of which material is considered.

Significant departures from the form of Equation (1) appear only at high neutron energies and will not influence the behavior of reactors of reasonable size.

GENERAL EQUATIONS

The general equation for neutron flux slowing-down, diffusion, and weak capture may be written

$$\frac{\partial}{\partial u}\xi(\vec{x})\,\Sigma_s(u,\vec{x})\phi(u,\vec{x}) = \nabla\cdot D(u,\vec{x})\nabla\phi(u,\vec{x}) - \Sigma_c(u,\vec{x})\phi(u,\vec{x}) \tag{2}$$

where ξ is the moderation factor,
Σ_s is the scattering macroscopic cross-section,
D the diffusion coefficient,
and Σ_c the capture cross-section.

With Equation (1) this equation becomes

$$\xi(\vec{x})\,\Sigma_{s\,th}(\vec{x})\frac{\partial}{\partial u}f(u)\phi(u,\vec{x}) = \frac{1}{f(u)}\nabla\cdot D_{th}(\vec{x})\nabla\phi(u,\vec{x}) - \Sigma_c(u,\vec{x})\phi(u,\vec{x})\,. \tag{3}$$

With the substitutions

$$t = \int_0^u \frac{1}{f^2(u')}\,du' \tag{4}$$

and

$$q = f\phi\,, \tag{5}$$

Equation (3) takes the form

$$\xi(\vec{x})\Sigma_{s\,th}(\vec{x})\frac{\partial}{\partial t}q(t,\vec{x}) = \nabla\cdot D_{th}(\vec{x})\nabla q(t,\vec{x}) - f(t)\Sigma_c(t,\vec{x})q(t,\vec{x})\,. \tag{6}$$

To within dimensional factors, t in Equation (4) has the form of a Fermi age for which q is the corresponding slowing-down density. Equation (6) is the slowing-down-density equation generalized for a space-dependent composition.

The only feature of Equation (6) preventing an analytical solution is the empirical function of t in the last term. The program below will take the following steps:

1. Replace the t dependence in the empirical function by an undetermined effective average value.
2. Solve Equation (6) analytically and find what spatial distributions of materials lead to flat flux and power.
3. Determine the effective constants under the assumed condition of flat flux.

The last step of the procedure will also yield a criticality value k for the reactor.

While the program outlined lacks rigor, it certainly should make vivid the qualitative features of a reflector-core reactor system. That good quantitative results are given as well may be checked by standard numerical methods of reactor calculation and finally by actual zero-power experiments.

Suppose

$$f(t)\,\Sigma_c(t,\vec{x}) = C^2(\vec{x}) \tag{7}$$

and let

$$\xi(\vec{x})\,\Sigma_{s\,th}(\vec{x}) = \omega(\vec{x})\,. \tag{8}$$

Suppressing the subscript of D_{th}, Equation (6) becomes

$$\nabla\cdot D(\vec{x})\nabla q(\vec{x},t) - C^2(\vec{x})q(\vec{x},t) = \omega(\vec{x})\frac{\partial}{\partial t}q(\vec{x},t), \tag{9}$$

which has solutions of the form

$$q_n(\vec{x},t) = q_n(\vec{x})\exp(-B_n^2 t), \tag{10}$$

where $q(\vec{x})$ satisfies the eigenvalue equation

$$\nabla\cdot D(\vec{x})\nabla q_n - C^2(\vec{x})q_n = -B_n^2\,\omega(\vec{x})q_n \tag{11}$$

with the familiar homogeneous boundary conditions. It is easily shown that solutions of Equation (11) have the weighted orthogonality property,

$$\int_v d^3x\,\omega(\vec{x})\,q_m(\vec{x})\,q_n(\vec{x}) = O \quad (m \neq n) \tag{12}$$

where the integral extends over the entire volume of the reactor. The most general solution of Equation (9) is

$$q(\vec{x},t) = \sum_n a_n\, q_n\,(\vec{x})\,\exp(-B_n^2 t). \quad (13)$$

If all neutrons are born at $t = O$ with a source distribution giving

$$q(\vec{x},O) = S(\vec{x}) \quad (14)$$

then Equations (12) and (13) give

$$a_n = \frac{\int_v d^3x\, \omega(\vec{x})\, S(\vec{x})\, q_n(\vec{x})}{\int_v d^3x\, \omega(\vec{x})\, q_n^2(\vec{x})} \quad (15)$$

which, substituted into Equation (13), completes the formal solution to the slowing-down problem.

The eigenvalues B_n^2 may be expected to increase rapidly with n. Hence, as t increases, the higher terms in Equation (13) become small compared to the first term. In practical cases, when a t has been reached at which fission becomes important, the higher terms are negligible and the solution (13) reduces to

$$q(x,t) = \frac{\int_v d^3x'\omega(\vec{x}')\, S(\vec{x}')\, q_0(\vec{x}')}{\int_v d^3x'\omega(\vec{x}')\, q_0^2(\vec{x}')}\, q_0(\vec{x})\exp(-B_0^2 t). \quad (16)$$

Except for the effects of thermal-neutron diffusion, the power and flux of a uniform core will be flat if $q_0(\vec{x})$ can be made constant throughout the core region. If this can be done, then we may have

$$S(\vec{x}) = \begin{cases} S & \text{in core} \\ O & \text{in reflector} \end{cases} . \quad (17)$$

For a reactor of uniform core and uniform reflector, let

$$\omega(\vec{x}) = \begin{cases} \omega_c & \text{in core} \\ \omega_R & \text{in reflector} \end{cases} , \quad (18)$$

$$D(\vec{x}) = \begin{cases} D_c & \text{in core} \\ D_R & \text{in reflector} \end{cases} , \quad (19)$$

and

$$C^2(\vec{x}) = \begin{cases} C^2 & \text{in core} \\ O & \text{in reflector} \end{cases} . \quad (20)$$

Then in the core Equation (11) becomes

$$D_c\,\nabla^2 q_0 - C^2 q_0 = -\omega_c B_0^2 q_0 . \quad (21)$$

Now the eigenvalue B_0^2 depends on the positions of the boundaries of the reflector. If these can be chosen so that

$$B_0^2 = \frac{1}{\omega_c}\, C^2 \quad (22)$$

then Equation (21) becomes

$$\nabla^2 q_0\,(\vec{x} \text{ in core}) = O \quad (23)$$

which permits the solution

$$q_0\,(\vec{x} \text{ in core}) = \text{const.} \quad (24)$$

which yields a reactor with flat flux and power.

In the reflector region Equation (11) becomes

$$D_R\,\nabla^2 q_0 + \omega_R B_0^2 q_0 = O, \quad (25)$$

which is easily solved for specified geometries, with the boundary conditions

$$q_0 = O \text{ at extrapolated exterior boundary} \quad (26)$$

$$\frac{\partial q_0}{\partial n} = O \text{ at reflector/core interface (from current-conservation)}. \quad (27)$$

The eigenvalue B_0^2 is a function of the thickness of the reflector. Conversely, the reflector thickness may be determined as a function of B_0, which in turn is determined from the core composition by Equation (22).

The effective capture constant C_0^2 has still to be determined. This may be done by noting that in the core, where the flux is flat, the space differentiations drop out of Equation (3), giving the infinite reactor equation

$$\omega_c \frac{\partial}{\partial u} f(u)\,\phi(u) = -\,\Sigma_c(u)\,\phi(u) \quad (28)$$

which yields the thermal flux

$$\phi(u_{th}) = \phi(0)\,\frac{f(0)}{f(u_{th})}\,\exp\left(-\frac{1}{\omega_c}\int_0^{u_{th}} du\,\frac{\Sigma_c(u)}{f(u)}\right). \quad (29)$$

Hence the simplification of Equation (7) will yield the correct thermal flux if

$$\int_0^{u_{th}} du\,\frac{\Sigma_c(u)}{f(u)} = \int_0^{u_{th}} du\,\frac{C^2}{f^2(u)} \quad (30)$$

or

$$C^2 = \frac{\int_0^{u_{th}} du\,\frac{\Sigma_c(u)}{f(u)}}{\int_0^{u_{th}} du\,\frac{1}{f^2(u)}} . \quad (31)$$

This numerically evaluated value of C^2 may be used to determine the value of B_0^2 in Equation (22). Evaluation of the reflector thickness from Equation (25) then completes the design of the flat-flux reactor.

CRITICALITY CALCULATION

The criticality k may also be evaluated along the lines of the previous paragraph. It has been mentioned that typically the higher buckling modes $q_n[n \neq O]$ have dropped out before a generation of neutrons start to cause fission. The principal mode q_0 is established by the diffusion of fast neutrons from the core to the reflector. The proportion of neutrons which does not escape in this fast diffusion will be given by the ratio of $q(\vec{x}$ in core, $t \to O)$ in Equation (16), to the value of S, the initial flux in the core. If the flux is flat, the nonleakage ratio given by Equation (16) is

$$\Lambda = \frac{q(\vec{x} \text{ in core, O})}{S(\vec{x} \text{ in core})} = \frac{1}{1 + \frac{\omega_R}{\omega_c}\frac{1}{v_c}\int_R d^3x\left(\frac{q_0(\vec{x})}{q_0(\text{core})}\right)^2} \tag{32}$$

where v_c is the volume of the core and the remaining unperformed integration extends over the reflector region only. After the initial fast leakage is complete there is no further leakage from the core, since the flux is flat there, and the subsequent history of the neutron economy proceeds exactly as in an infinite reactor. Hence, the criticality is given by

$$k = \Lambda k_\infty \tag{33}$$

The nonleakage factor Λ may be evaluated by Equation (32), and k_∞ by well-known standard means.

Several points should be made here. The weak capture Equation (2) was used primarily for pedagogical purposes and, if material composition demands a more stringent treatment of neutron slowing-down, then a more accurate evaluation of C^2 may be used, the criterion still being that the thermal flux must be given properly. Likewise, the approximation of diffusion theory in the core was more apparent than real. If the flux is flat, there will be no important transport effects in the core, and indeed the core diffusion coefficient D_C dropped out of the theory at Equation (23). Furthermore, the distribution of the fast-neutron source over high energies will not critically change the results above, which followed from a simplified mono-energetic source, so long as the principal mode establishes itself before important fission occurs. Equation (32) shows that the fundamental consideration is simply what fraction of neutrons soak into the reflector and what fraction spend their lives in the core. Finally, no account has been taken of the diffusion of thermal neutrons. Back diffusion of thermal neutrons from the reflector under some circumstances will lead to a "thermal spike" of fission density near the core interface. While this tends to increase k, it is undesirable in a high-power reactor where the entire core is running under environmental conditions near the failure point. Furthermore, the cleanliness of the theory above is spoiled by this effect. In a following paper it will be shown how thermal-neutron diffusion may be eliminated and why in a periodic core of fuel and moderator regions the thermal-spike effect may be entirely absent, validating the present theory.

SPECIFIC CASES

Specializing the general theory to the case of a finite cylindrical reactor poses no great difficulties. For example, the top reflector thickness L_1 is obtained by solving Equation (25) in slab geometry with boundary condition, Equation (27), and then demanding that the boundary condition, Equation (26), also be satisfied: Equation (26) becomes

$$\cos\sqrt{\omega_R/D_R}\; B_0 L_1 = O \tag{34}$$

or

$$L_1 = \frac{\pi}{2}\sqrt{D_R/\omega_R}\;\frac{1}{B_0} \tag{35}$$

for the thickness (including extrapolation length). In the side reflector Equation (25) solves in terms of Bessel functions:

$$q_0(r) = J_0(\sqrt{\omega_R/D_R}\; B_0 r) + \alpha Y_0(\sqrt{\omega_R/D_R}\; B_0 r) \tag{36}$$

where the constant α is determined by Equation (27):

$$\alpha = \frac{J_1(\sqrt{\omega_R/D_R}\;B_0R_0)}{Y_1(\sqrt{\omega_R/D_R}\;B_0R_0)}\;. \tag{37}$$

Here R_0 is the radius of the core. The exterior radius R_1 may now be found from Equation (26) which becomes the transcendental equation

$$\frac{J_0(\sqrt{\omega_R/D_R}\;B_0R_1)}{Y_0(\sqrt{\omega_R/D_R}\;B_0R_1)} = \alpha \tag{38}$$

to be solved for R_1. These steps may be systematized in the following way: Let

$$\sqrt{\omega_R/D_R}\;B_0R_0 = x, \quad \text{and} \quad \sqrt{\omega_R/D_R}\;B_0R_1 = y. \tag{39}$$

Then Equation (37) becomes

$$\alpha(x) = \frac{J_1(x)}{Y_1(x)}\,, \tag{40}$$

determining $\alpha(x)$ from known x, and Equation (38) becomes

$$\frac{J_0(y)}{Y_0(y)} = \alpha(x) \tag{41}$$

determining implicitly y from known x. The relationship (41) may be solved numerically and tabulated, once and for all, for

$$y = \Theta(x). \qquad (42)$$

The outer radius of the reactor is then given by

$$R_1 = \Theta(\sqrt{\omega_R/D_R}\; B_0 R_0)\, \sqrt{D_R/\omega_R}\; \frac{1}{B_0}. \qquad (43)$$

Thus, $\Theta(x)$ plays the same role in Equation (43) as did $\frac{\pi}{2}$ in the end-reflector solution (35).

For the finite-cylinder reactor, the reflector integral in Equation (32) may also be evaluated, giving the fast leakage factor

$$\Lambda = \frac{1}{1 + \frac{\omega_R}{\omega_c}\left[\frac{L_1}{L_0} + \frac{R_1^2}{R_0^2}\,\Phi(\sqrt{\omega_R/D_R}\; B_0 R_0) - 1\right]} \qquad (44)$$

where

$$\Phi(x) = \left(\frac{J_0(x) - \alpha(x)\;Y_0(x)}{J_1[\Theta(x)] - \alpha(x)\,Y_1[\Theta(x)]}\right)^2. \qquad (45)$$

Equations (35), (43), and (44) (together with k_∞) give explicit means for calculating the reflector thickness and criticality of a finite cylindrical flat-flux reactor with a specified uniform core.

APPLICATION

The method above has been used at Los Alamos, with excellent success, in the preliminary designing of hydrogen-moderated flat-flux reactors with cores containing molybdenum or tungsten and reflectors of graphite or beryllium. In a mechanized version for the IBM 704, a Newton's-method routine enables the machine to find a critical reactor by adjusting either uranium loading or moderator density in an otherwise specified core. At the rate of about one iteration per second, several reactors per minute can be calculated in parameter studies, each one critical and with a flat flux and power profile.

The theory presented here essentially stipulates that proper choice of reflector dimensions permits buckling leakage in the reflector to proceed at the same rate as capture proceeds in the core. This allows a fixed flux profile to be established and maintained at all lethargies without net neutron flow across the core/reflector interface. For actual materials Equation (7) is not satisfied, however, and the depletion rates in core and reflector will consequently get out of step. At lethargies where capture in the core is higher than the value Equation (7) would give, the flux level in the center of the core will drop, and neutron currents will flow in from the reflector. When capture in the core is low, the opposite will happen. But the value of C^2 is such that these two situations will alternate as lethargy increases, and the fission density will tend to average out these departures from the idealized theory. The analysis of these flux nonuniformities is beyond the reach of the simple analytic theory, but may be studied by standard numerical reactor codes. Figures 1 and 2 show the results of an age-diffusion numerical integration on an IBM 704 using the Los Alamos "Fire" code with 18 energy groups. The reactor, very similar to one whose specifications are given below, had a hydrogen-moderated core containing molybdenum, and a graphite reflector.

Figure 1 shows the flux profile in its initial stages—fast leakage is nearing completion at ϕ_5 (the lethargy groups are not evenly spaced). At ϕ_5 the core capture is higher than its "balance" value given by Equation (7) and consequently the flux is sagging at the edges. At ϕ_6 and ϕ_7 (Fig. 2) this situation is reversed. At ϕ_8 in Fig. 2 the situation is reversed again, leading finally to a quite flat flux in the core by the end of the lethargy range. The fission distribution is seen to be flat to within about ±5%.

The reactor design input into the numerical calculation was obtained from the analytic flat-flux method of this paper. However, the reactor had to be volumetrically scaled to an "equivalent sphere" as the "Fire" code will not handle finite cylinders. The less reliable standard molybdenum cross-sections of the "Fire" code also may have influenced the results somewhat.

Figure 3 shows the experimentally determined fission profile radially in a similar reactor, determined by the residual activity of U^{235} foils placed in a low-power assembly. The saw-tooth effect is probably due in part to the somewhat lumped core configuration. Effective lumped cross-sections were used in the flat-flux routine for the design of this reactor.

Table I and II below compare the results of the flat-flux design procedure with the outcomes of two typical critical assemblies based on such pre-design. Both were near right-circular cylinders

TABLE I

A Flat-Flux Reactor Containing Molybdenum

	Flat-flux calculations	Critical experiment
Core, volume, liters	331.0	343.4
Side reflector thickness (C), cm	31.0	30.5
Average molar densities, atoms/cm³ × 10^{-20}		
Mo	47.0	47.0
H	83.7	78.0
U	2.98	2.987
Criticality	$k = 1$	$k = 1$

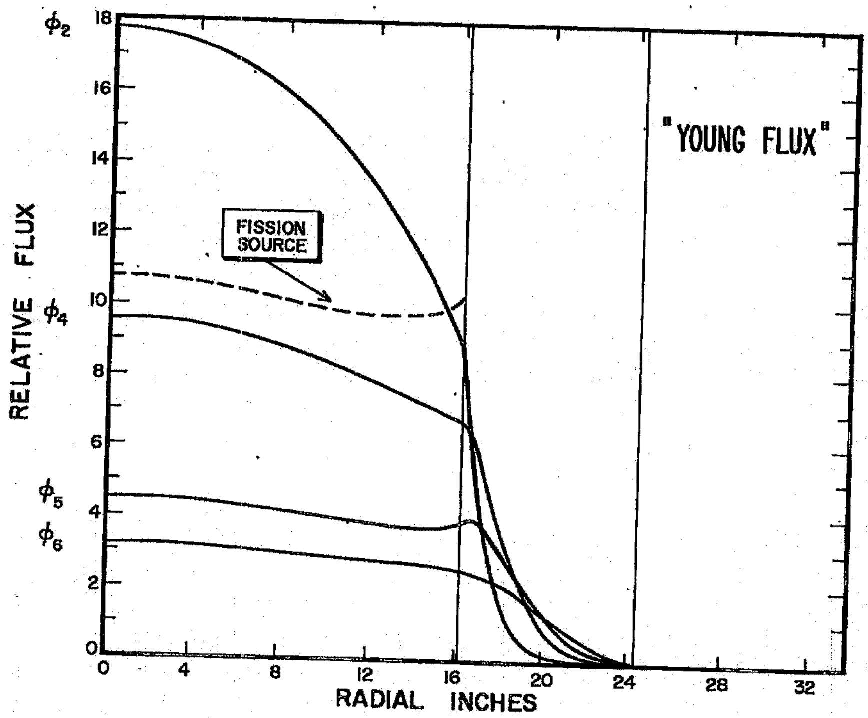

Fig. 1. Machine calculation of early neutron diffusion in the reactor of Table I.

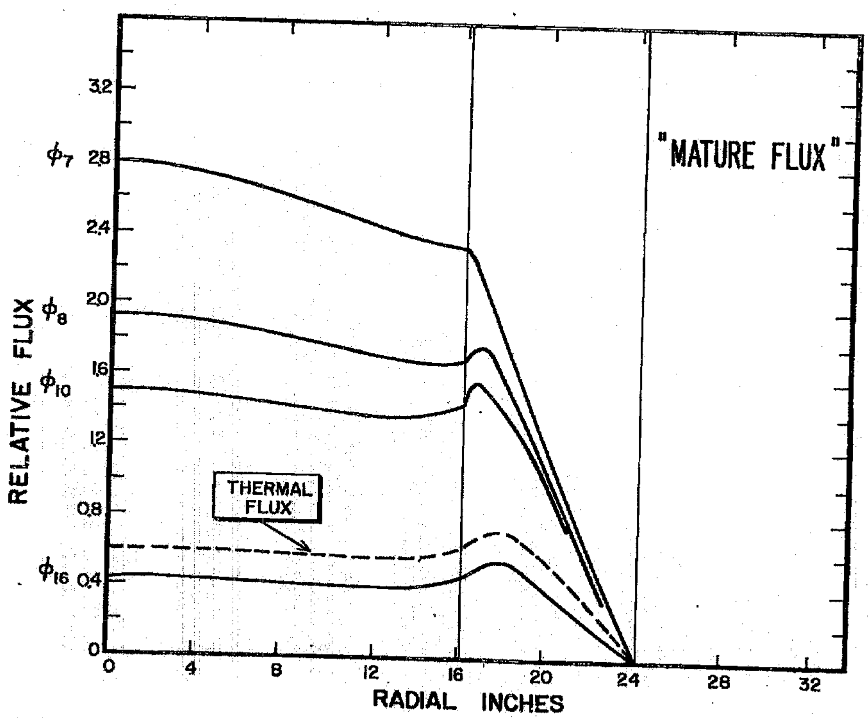

Fig. 2. Subsequent diffusion, after flux in reflector has been established.

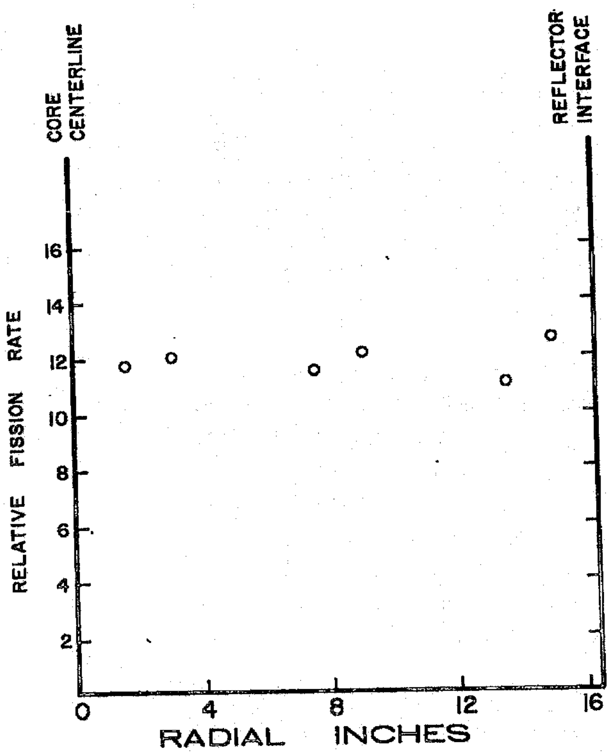

Fig. 3. Foil measurements of fission level in reactor of Table I.

($L_0 = 2R_0$). The H/U ratios were rather different. Table I shows the reactor described above, with graphite reflector and molybdenum in the core, whereas the reactor of Table II is beryllium-reflected with a core containing tungsten. Both design calculations were performed with input cross - sections not empirically adjusted, and before the corresponding critical assemblies were constructed.

The experiments were brought to criticality by the addition of U^{235} foils. Discrepancies between theory and experiment in other design features are due to the constraints of modular construction in the Los Alamos Honeycomb critical assembly machine, which was used to obtain these results. The excellent agreement in the two cases shown is typical of results obtained for a fairly wide variety of similar reactors.

TABLE II

A Flat-Flux Reactor Containing Tungsten

	Flat-flux calculation	Critical experiment
Core volume, liters	272.5	244.2
Side reflector thickness (Be), cm	23.56	22.86
Average molar densities, atoms/cm^3 × 10^{-20}		
W	13.49	13.07
H	210.13	222.1
U	1.747	1.688
Criticality	$k = 1$	$k = 1$

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NOTE ADDED IN PROOF

A more primitive but much more detailed development of this material may be found in Los Alamos Scientific Laboratory Report LA-2091-del, recently declassified.